

Solving Literal Equations

Equations and formulas that have **several variables** (letters) are called **literal equations**. Your job, usually, will be to solve the equation for one of the variables. In other words, move all the letters away from the one you are solving for. They go to the other side of the equal sign so that the one variable you are solving for stands **alone**.

1st REVERSE PEMDAS
2nd
SA **DM** EP

In examples #1 - 3 solve for x:

<p>1a) $\frac{4x}{4} = \frac{12}{4}$ → like terms</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = 3$ </div>	<p>1b) $\frac{ax}{a} = \frac{b}{a}$ → not like term $a \neq 0$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = \frac{b}{a}$ </div>
<p>2a) $(x) + 4 = 7$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = 3$ </div>	<p>2b) $x + a = b$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = b - a$ </div> <p style="font-size: small;">• Can also write as $x = -a + b$ NOT $x = a - b$</p>
<p>3a) $3x + 2 = 5$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $x = 1$ </div>	<p>3b) $ax + c = d$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $a = \frac{d - c}{a}$ </div> <p style="text-align: right; font-size: small;">$a \neq 0$</p>

4) Solve for x:

$ax - g = 8g$

$x = \frac{9g}{a}$

a ≠ 0
a ≠ 0

5) Solve for w:

$r = 5(w + x)$

First Distribute $w = \frac{r}{5} - \frac{5x}{5}$

$w = \frac{r - 5x}{5}$

can also be $w = \frac{1}{5}(r - 5x)$
B/c it is $\frac{r}{5} - \frac{5x}{5}$

6) Brandon knows that his train route from Illinois to Tennessee is 430 miles long. He also knows that **Distance = rate • time** ($D = rt$)

(a) Solve for t in the formula $D = rt$

$t = \frac{D}{r}$



$r \neq 0$

(b) How long will his route take if he averages a speed of 50 mi/hr?

$t = \frac{D}{r}$

$t = \frac{430}{50}$

$t = 8.6$

Key

if you have a fraction in an equation, you clear get rid of it 1st by multiplying by the reciprocal

More Examples: Solve for the stated values in the given equations.

1) $4x - a = c$ for x

$$\begin{array}{r} +a +a \\ 4x = c+a \\ \hline \frac{4x}{4} = \frac{c+a}{4} \end{array}$$

$$x = \frac{c+a}{4}$$

2) $I = PRT$ for T *Interest*

$$T = \frac{I}{PR}$$

$PR \neq 0$ or $R \neq 0$

3) $a(x+b) = c$ for x

$$\begin{array}{r} a(x+b) = c \\ -ab -ab \\ \hline ax = c-ab \\ \hline \frac{ax}{a} = \frac{c-ab}{a} \end{array}$$

$$x = \frac{c-ab}{a} \text{ or } x = \frac{c}{a} - b$$

$a(x+b) = c$
 $\frac{a(x+b)}{a} = \frac{c}{a}$
 $x+b = \frac{c}{a}$
 $-b -b$
 $x = \frac{c}{a} - b$

4) $\frac{2}{3}y = \frac{3}{2}x$ for y

$$y = \frac{3}{2}x$$

5) $A = \frac{1}{2}bh$ for h *Area of a Triangle*

$$2A = bh$$

$$h = \frac{2A}{b}$$

$b \neq 0$

6) $5b + 3a = 7b - 4a$ for a

$$\begin{array}{r} +4a +4a \\ 5b+7a = 7b \\ -5b -5b \\ \hline 7a = 2b \end{array}$$

$$a = \frac{2b}{7} \text{ or } a = \frac{2}{7}b$$

7) $8x - 3y = 4x - 3y$ for x

$$\begin{array}{r} -4x -4x \\ 4x - 3y = -3y \\ +3y +3y \\ \hline 4x = 0 \\ \hline \frac{4x}{4} = \frac{0}{4} \\ x = 0 \end{array}$$

Some cases $\frac{1}{3}(4x-2f)$ so multiply both by 3 (reciprocal)

8) $C = 2\pi r$ for r *Circumference*

$$r = \frac{C}{2\pi}$$

multiply both sides by the denominator to get rid of it

9) $D = \frac{m}{v} \times V$ for v *Density* $(\frac{m}{v})D = \frac{m}{v} \times V$

$$\begin{array}{l} \frac{vD}{D} = \frac{m}{D} \text{ or } vD = \frac{m}{D} \\ \hline v = \frac{m}{D} \end{array}$$

10) $\frac{4x-2f}{5} = a$ for x

binomial
 $4x - 2f = 5a$
 $+2f +2f$
 $4x = 5a + 2f$
 $\frac{4x}{4} = \frac{5a + 2f}{4}$
 $x = \frac{5a + 2f}{4}$

11) $C = \pi r^2$ for r

$$r = \sqrt{\frac{C}{\pi}}$$

take the square root of r b/c it's the inverse of squaring

12) $d = xy^2$ for y

$$y = \sqrt{\frac{d}{x}}$$

$x \neq 0$

13) $\frac{z^2}{x} = \frac{4y}{x}$ for z

$$\begin{array}{l} \frac{z^2}{x} = \frac{4y}{x} \\ \sqrt{z^2} = \sqrt{\frac{4y}{x}} \\ \hline z = \sqrt{\frac{4y}{x}} \end{array}$$

14) $V = \pi r^2 h$ for h & then for r *Volume of cylinder*

$$\begin{array}{l} V = \pi r^2 h \\ \frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \\ \hline h = \frac{V}{\pi r^2} \\ \hline r^2 = \sqrt{\frac{V}{\pi h}} \\ \hline r = \sqrt{\frac{V}{\pi h}} \end{array}$$

15) $\frac{ax}{y} = \frac{yb}{a}$ for b $(a)(x) = (\frac{y}{a}b)(a)$

$$\begin{array}{l} b = \frac{ax}{y} \text{ or } ax = \frac{y}{y}b \\ \hline b = \frac{ax}{y} \end{array}$$

$$(16) r = \left(\frac{1}{3}xy^2\right)^{\frac{1}{3}} \text{ for } y$$

$$3r = \frac{xy^2}{3}$$

$$y^2 = \frac{3r}{x}$$

$$y = \sqrt{\frac{3r}{x}} \quad x \neq 0$$

$$17) bc + d = f \text{ for } c$$

$$\frac{bc + d = f}{-d - d}$$

$$\frac{bc = f - d}{b}$$

$$c = \frac{f - d}{b} \quad b \neq 0$$

$$18) m = 2(x + n) \text{ for } x$$

$$\frac{m = 2x + 2n}{-2n \quad -2n}$$

$$\frac{2x = m - 2n}{2}$$

$$x = \frac{m - 2n}{2} \quad \text{or} \quad x = \frac{m}{2} - n$$

OR ~
 $m = 2(x + n)$
 $\frac{m}{2} = \frac{2(x + n)}{2}$
 $\frac{m}{2} = x + n$
 $x = \frac{m}{2} - n$

$$19) ax - by = c \text{ for } x$$

$$+by \quad +by$$

$$\frac{ax = c + by}{a}$$

$$x = \frac{c + by}{a} \quad a \neq 0$$

$$20) P = 2L + 2W \text{ for } W \text{ \textit{Perimeter of a rectangle}}$$

$$\frac{-2L \quad -2L}{-2L \quad -2L}$$

$$\frac{2W = P - 2L}{2}$$

$$W = \frac{P - 2L}{2} \quad \text{or } W = \frac{P}{2} - L$$

$$21) abx - d = 5d \text{ for } x$$

$$\frac{+d \quad +d}{abx = 6d}$$

$$x = \frac{6d}{ab}$$

$$a \neq 0 \text{ or } b \neq 0$$

9
 $(22) C = \frac{9}{5}(F - 32) \text{ for } F$ *must be in C*
it is a binomial

$$\frac{9}{5}C = F - 32$$

$$F = \frac{9}{5}C + 32$$

$$23) \text{ For } x: a + 12x = m$$

$$\frac{-a \quad -a}{12x = m - a}$$

$$\frac{12x = m - a}{12}$$

$$x = \frac{m - a}{12}$$

$$24) 4x + 4b = 12 \text{ for } x$$

$$\frac{-4b \quad -4b}{4x = 12 - 4b}$$

$$\frac{4x = 12 - 4b}{4}$$

$$x = \frac{12 - 4b}{4} \rightarrow x = \frac{12}{4} - \frac{4b}{4}$$

$$x = 3 - b$$

$$25) H = \frac{cd}{c} \text{ for } d$$

$$\frac{c \quad c}{d = \frac{H}{c}}$$

$$d = \frac{H}{c}$$

$$c \neq 0$$

$$26) Z = 3c + Pf \text{ for } P$$

$$\frac{-3c \quad -3c}{Pf = Z - 3c}$$

$$\frac{Pf = Z - 3c}{f}$$

$$P = \frac{Z - 3c}{f} \quad f \neq 0$$

$$27) D = crt + c \text{ for } t$$

$$\frac{-c \quad -c}{D - c = crt}$$

$$t = \frac{D - c}{cr}$$

$$\text{or } t = \frac{D}{cr} - \frac{1}{r}$$

c \neq 0
OR
r \neq 0

$$28) Kx + y = g \text{ for } x$$

$$\frac{-y \quad -y}{Kx = g - y}$$

$$x = \frac{g - y}{K}$$

$$K \neq 0$$

$$(29) C = \left(\frac{ab}{2}\right)^{\frac{1}{3}} \text{ for } b$$

$$\frac{2C = \frac{ab}{2}}{a}$$

$$b = \frac{2C}{a}$$

$$a \neq 0$$

$$30) V = \left(\frac{1}{4}\pi y^2 z\right)^{\frac{1}{3}} \text{ for } y$$

$$\frac{4V = \frac{1}{4}\pi y^2 z}{\frac{1}{4}\pi z}$$

$$y^2 = \frac{4V}{\frac{1}{4}\pi z}$$

$$y = \sqrt{\frac{4V}{\frac{1}{4}\pi z}}$$

$$z \neq 0$$

31) $2y + 3x = 18$ for y

$$\frac{2y}{2} = \frac{18 - 3x}{2}$$

$$y = \frac{18 - 3x}{2}$$

or $y = 9 - \frac{3}{2}x$

32) $\frac{2x - 8y}{6} = (c, d)$ for x
must have (c) B/c Numerator is a binomial

$$2x - 8y = 6c$$

$$+8y \quad +8y$$

$$2x = 6c + 8y$$

$$\frac{2x}{2} = \frac{6c + 8y}{2}$$

$$x = \frac{6c + 8y}{2} \rightarrow x = 3c + 4y$$

33) $a = \frac{2b}{c^3}$ for c

$$\frac{c^3 a}{a} = \frac{2b}{a}$$

$$c^3 = \sqrt[3]{\frac{2b}{a}}$$

$$c = \sqrt[3]{\frac{2b}{a}}$$

Take the 3rd of c b/c it is the inverse of cubing c

must have (c) B/c numerator is a binomial

34) $m = \frac{xy}{2}$ for x

$$\frac{2m}{y} = \frac{xy}{y}$$

$$x = \frac{2m}{y}$$

$y \neq 0$

35) $\frac{3}{5}x = \frac{y+2}{2}$ for y

$$\frac{3}{5}x = \frac{y+2}{2}$$

$$y = \frac{3}{5}x - 2$$

or $e(x+c) = 2b$
 $ex + ec = 2b$
 $-ec \quad -ec$
 $ex = 2b - ec$
 $\frac{ex}{e} = \frac{2b - ec}{e}$
 $x = \frac{2b - ec}{e}$

36) $\frac{e(x+c)}{b} = 2b$ for x

$$e(x+c) = 2b$$

$$ex + ec = 2b$$

$$-ec \quad -ec$$

$$ex = 2b - ec$$

$$\frac{ex}{e} = \frac{2b - ec}{e}$$

$$x = \frac{2b - ec}{e}$$

$e \neq 0$

37) $\frac{3(x-k)}{w} = 4w$ for x

$$3(x-k) = 4w$$

$$3x - 3k = 4w$$

$$+3k \quad +3k$$

$$3x = 4w + 3k$$

$$\frac{3x}{3} = \frac{4w + 3k}{3}$$

$$x = \frac{4w + 3k}{3}$$

38) $a(x+b) - c = d$ for x

$$a(x+b) = d + c$$

$$ax + ab = d + c$$

$$-ab \quad -ab$$

$$\frac{ax}{a} = \frac{d + c - ab}{a}$$

$$x = \frac{d + c - ab}{a}$$

$a \neq 0$

39) $F = \frac{9}{5}c + 32$ for c

$$F - 32 = \frac{9}{5}c$$

$$\frac{5}{9}(F - 32) = \frac{5}{9} \cdot \frac{9}{5}c$$

$$c = \frac{5}{9}(F - 32)$$

Fahrenheit formula
Don't get rid of fraction 1st b/c reverse of 32 is 15

40) Solve $4 = 2m - 5n$ for m. What are the values of m when $n = -2, 0$ and 2 .

$$2m = 5n + 4$$

$$\frac{2m}{2} = \frac{5n + 4}{2}$$

$$m = \frac{5n + 4}{2}$$

$n = 2$

$$m = \frac{5(2) + 4}{2}$$

$$m = \frac{10 + 4}{2}$$

$$m = \frac{14}{2}$$

$$m = 7$$

$n = 0$

$$m = \frac{5(0) + 4}{2}$$

$$m = \frac{0 + 4}{2}$$

$$m = \frac{4}{2}$$

$$m = 2$$

$n = -2$

$$m = \frac{5(-2) + 4}{2}$$

$$m = \frac{-10 + 4}{2}$$

$$m = \frac{-6}{2}$$

$$m = -3$$