

Analyzing Solutions

I. Possible Solutions of One-Variable Equations

Until now, when you have solved equations, there has been only one solution. It is also possible for a one-variable equation to have no solutions or infinitely many solutions.

EXAMPLE 1 One-Variable Equations with No Solutions

Solve the equation, and interpret the result.

$$2x - 4 = 2(x - 1) + 2$$

$$2x - 4 = 2(x - 1) + 2$$

$$2x - 4 = 2x$$

2x - 2 + 2 is 2x.

$$\begin{array}{r} +4 \\ 2x - 4 = 2x \\ \hline 2x = 2x + 4 \end{array}$$

Add 4 to both sides.

$$\begin{array}{r} -2x \\ 2x = 2x + 4 \\ \hline 0 = 4 \end{array}$$

Subtract 2x from both sides.

This is a false statement.

Because $0 = 4$ is never a true statement, the equation can never be true for *any value* of x . There is no solution.

EXAMPLE 2 One-Variable Equations with Infinitely Many Solutions

Solve the equation, and interpret the result.

$$-4x + 3(x - 1) = -(x + 3)$$

$$-4x + 3(x - 1) = -(x + 3)$$

$$-4x + 3x - 3 = -x - 3$$

Apply the Distributive Property.

$$-x - 3 = -x - 3$$

Simplify.

$$\begin{array}{r} +3 \\ -x - 3 = -x - 3 \\ \hline -x = -x \end{array}$$

Add 3 to both sides.

$$-x = -x$$

$$\begin{array}{r} +x \\ -x = -x \\ \hline 0 = 0 \end{array}$$

Add x to both sides.

This is a true statement.

Because $0 = 0$ is always a true statement, the equation is always true for *any value* of x . There are *infinitely many* solutions.

Result	What does this mean	How many solutions
$x = a$	When the value of x is a , the equation is a true statement	1
$a = a$	Any value of x makes the equation a true statement.	Infinitely many
$a = b$	There is no value of x that makes the equation true.	0

Solve the equation, and interpret the result. Tell whether each equation has one, zero, or infinitely many solutions.

1) $6 + 3x = x - 8$

2) $4x + 2 = 4x - 5$

3) $4x - 3 = 2x + 13$

4) $4x - 5 = 2(2x - 1) - 3$

5) $8x + 4 = 4(2x + 1)$

6) $7x + 2(x - 2) = 3(3x + 4)$