Name Key Date_____

Mrs. Roumbos 8A Period_____

# Analyzing Solutions

## I. Possible Solutions of One-Variable Equations

Until now, when you have solved equations, there has been only one solution. It is also possible for a one-variable equation to have no solutions or infinitely many solutions.

### EXAMPLE 1 One-Variable Equations with No Solutions

**Solve the equation, and interpret the result.**

$2x - 4 = 2(x - 1) + 2$

$$\begin{aligned} 2x - 4 &= 2(x - 1) + 2 \\ 2x - 4 &= 2x && \text{2x − 2 + 2 is 2x.} \\ +4 & \quad +4 && \text{Add 4 to both sides.} \\ 2x &= 2x + 4 \\ -2x & \quad -2x && \text{Subtract 2x from both sides.} \\ 0 &\neq 4 && \text{This is a false statement.} \end{aligned}$$

Because $0 = 4$ is never a true statement, the equation can never be true for *any value* of $x$. There is no solution.

### EXAMPLE 2 One-Variable Equations with Infinitely Many Solutions

**Solve the equation, and interpret the result.**

$-4x + 3(x - 1) = -(x + 3)$

$$\begin{aligned} -4x + 3(x - 1) &= -(x + 3) \\ -4x + 3x - 3 &= -x - 3 && \text{Apply the Distributive Property.} \\ -x - 3 &= -x - 3 && \text{Simplify.} \\ +3 & \quad +3 && \text{Add 3 to both sides.} \\ -x &= -x \\ +x & \quad +x && \text{Add x to both sides.} \\ 0 &= 0 && \text{This is a true statement.} \end{aligned}$$

Because $0 = 0$ is always a true statement, the equation is always true for *any value* of $x$. There are *infinitely many* solutions.

| Result | What does this mean | How many solutions |
|---|---|---|
| x = a<br>ex X=5 | When the value of x is a, the equation is a true statement | 1 |
| a = a<br>ex 7=7 | Any value of x makes the equation a true statement. | Infinitely many<br>∞ |
| a = b<br>ex 6≠4 | There is no value of x that makes the equation true. | 0<br>Zero / None / No Solutions |

Can ask: (1) How many solutions? (2) What are the # of solutions

Must have variables (x)

Can't cancel variables only constants

0x 1x

(3) Solve the equation, and interpret the result. Tell whether each equation has one, zero, or infinitely many solutions.

1) $6 + 3x = x - 8$

D
C
M
S

$-x \quad -x$

$6 + 2x = -8$

$-6 \quad -6$

$\frac{2x}{2} = \frac{-14}{2}$

$x = -7$

one solution

2) $4x + 2 = 4x - 5$

D
C
M
S

$-4x \quad -4x$

$2 = -5$

No solutions

3) $4x - 3 = 2x + 13$

D
C
M
S

$-2x \quad -2x$

$2x - 3 = 13$

$+3 \quad +3$

$\frac{2x}{2} = \frac{16}{2}$

$x = 8$

One solution

4) $4x - 5 = 2(2x - 1) - 3$

D
C
M
S

$4x - 5 = 4x - 2 - 3$

$4x - 5 = 4x - 5$

$-4x \quad -4x$

$-5 = -5$

Infinitely many solutions

5) $8x + 4 = 4(2x + 1)$

D
C
M
S

$8x + 4 = 8x + 4$

$-8x \quad -8x$

$4 = 4$

Infinitely many solutions

6) $7x + 2(x - 2) = 3(3x + 4)$

D
C
M
S

$7x + 2x - 4 = 9x + 12$

$9x - 4 = 9x + 12$

$-9x \quad -9x$

$-4 = 12$

Zero/None/No solution