Name Key

Date__________

A; Algebra 1

Period__________

What is Set Notation?

VOCABULARY

A **set** is a collection of objects. The objects are called **elements** or members of the set.

Set *B* is a **subset** of set *A* if every element of *B* is an element of *A*. ex whole #'s are a subset of Integers

A **finite set** contains a limited number of elements, such as {1, 2, 3, 4}.

An **infinite set** contains an unlimited number of elements, such as {1, 2, 3, 4 . . .}.

The **empty set**, or **null set**, is a set that has no elements and is written ∅ or { }. {∅}

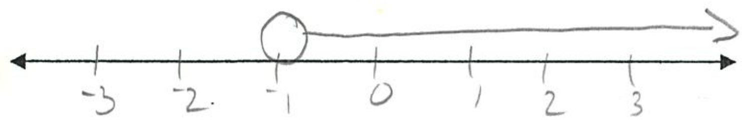
Review

An inequality is an algebraic sentence that includes an inequality symbol (<, >, ≤, ≥) It is a comparison of 2 quantities

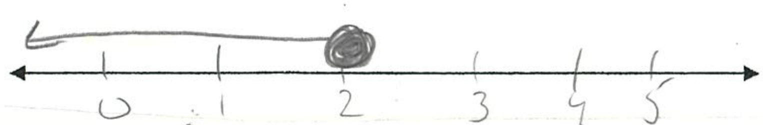
Graph:

x > -1

answers #'s



y ≤ 2



AND

#'s must satisfy both inequalities

answers

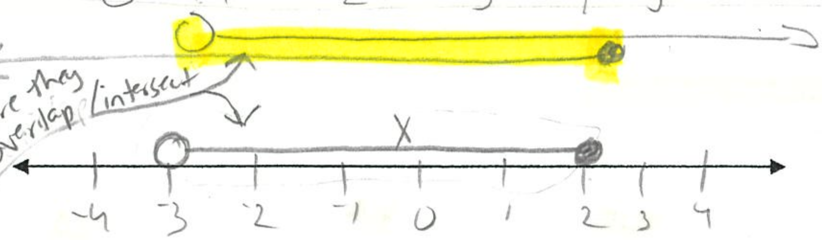
Compound inequality

-3 < x ≤ 2

x > -3 and x ≤ 2

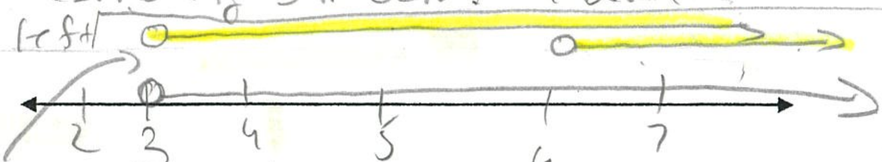
plot what they have in common

where they overlap / intersect



when the x is in between, the shading is in between and the arrows always point to the left

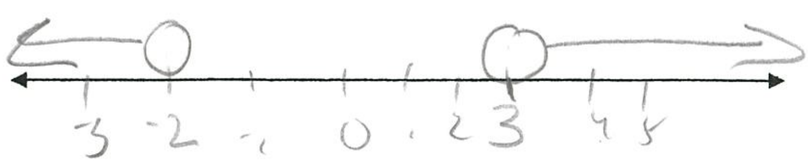
(x > 3) or (x > 6)



The answers (#'s) only have to satisfy one of the inequalities

graph the inequality that covers more #'s (Bigger one)

(x < -2) or (x > 3)



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KEY CONCEPT: Set Notation

1 *Words:* A is the set whose elements are 1, 2, 3, 4, 5.

2 *Roster form:* $A = \{1, 2, 3, 4, 5\}$

3 *Set-Builder notation:* $A = \{x \mid 1 \leq x \leq 5 \text{ and } x \text{ is an integer}\}$

4 Interval notation is used to describe a set that includes all numbers between two numbers.

Interval notation: $B = [1, 5] \leftarrow$ This means all *real* numbers between 1 and 5.

Use parentheses instead of brackets to exclude endpoint numbers.

$C = [1, 5) \leftarrow$ All numbers between 1 and 5 that includes 1 but not 5.

Elements of a set are written as $4 \in A$, which is read as "4 is an element of A."
Subsets are written $A \subseteq B$, which is read "A is a subset of B."

A **set** is a collection of unique elements. Elements in a set do not "repeat".

Methods of Describing Sets:
Sets may be described in many ways: by roster, by set-builder notation, by interval notation, by graphing on a number line, and/or by Venn diagrams.

By roster: A **roster** is a list of the elements in a set, separated by commas and surrounded by French curly braces.

Exclusive: not including ○

$\{2, 3, 4, 5, 6\}$	is a roster for the set of integers from 2 to 6, inclusive. Including ●
$\{1, 2, 3, 4, \ldots\}$	is a roster for the set of positive integers. The three dots indicate that the numbers continue in the same pattern indefinitely. (Those three dots are called an **ellipsis**.)
Rosters may be awkward to write for certain sets that contain an infinite number of entries.	

By set-builder notation: **Set-builder notation** is a mathematical shorthand for precisely stating all numbers of a specific set that possess a specific property.

$\mathbb{R}$ = real numbers; $\mathbb{Z}$ = integer numbers; $\mathbb{N}$ = natural numbers.

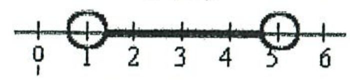
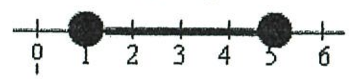
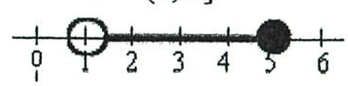
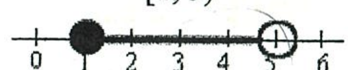
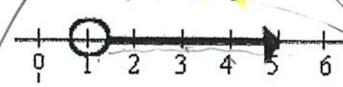
$\{x \in \mathbb{Z} \mid 2 \leq x \leq 6\}$	is set-builder notation for the set of integers from 2 to 6, inclusive. $\in$ = "is an element of" Z = the set of integers $\mid$ = the words "such that" The statement is read, "all x that are elements of the set of integers, such that, x is between 2 and 6 inclusive."
$\{x \in \mathbb{Z} \mid x > 0\}$	The statement is read, "all x that are elements of the set of integers, such that, the x values are greater than 0, positive." (The positive integers can also be indicated as the set Z^+.)
It is also possible to use a colon (:), instead of the $\mid$, to represent the words "such that". $\{x \in \mathbb{Z} \mid 2 \leq x \leq 6\}$ is the same as $\{x \in \mathbb{Z} : 2 \leq x \leq 6\}$	

By interval notation: An **interval** is a connected subset of numbers. **Interval notation** is an alternative to expressing your answer as an inequality. Unless specified otherwise, we will be working with real numbers.

When using interval notation, the symbol:
(means "not included" or "open". ○ $< >$
[means "included" or "closed". ● $\leq \geq$

7th	$2 \leq x < 6$	as an inequality.
9th	$[2, 6)$	in interval notation.

The chart below will show you all of the possible ways of utilizing interval notation.

Interval Notation: (description)	(diagram)
Open Interval: (a, b) is interpreted as $a < x < b$ where the endpoints are NOT included. (While this notation resembles an ordered pair, in this context it refers to the interval upon which you are working.)	(1, 5) 
Closed Interval: $[a, b]$ is interpreted as $a \leq x \leq b$ where the endpoints are included.	[1, 5] 
Half-Open Interval: $(a, b]$ is interpreted as $a < x \leq b$ where a is not included, but b is included.	(1, 5] 
Half-Open Interval: $[a, b)$ is interpreted as $a \leq x < b$ where a is included, but b is not included.	[1, 5) 
Non-ending Interval: (a, ∞) is interpreted as $x > a$ where a is not included and infinity is always expressed as being "open" (not included).	$(1, \infty)$ 
Non-ending Interval: $(-\infty, b]$ is interpreted as $x \leq b$ where b is included and again, infinity is always expressed as being "open" (not included).	$(-\infty, 5]$ 

* $+\infty$ and $-\infty$

must have ( )

*All: must be in numerical order
(read graph from left to right)

For some intervals it is necessary to use combinations of interval notations to achieve the desired set of numbers. Consider how you would express the interval "all numbers except 13".

As an inequality:	$x < 13$ or $x > 13$
In interval notation:	$(-\infty, 13) \cup (13, \infty)$
Notice that the word "or" has been replaced with the symbol "U", which stands for "union".	

Consider expressing in interval notation, the set of numbers which contains all numbers less than 0 and also all numbers greater than 2 but less than or equal to 10.

As an inequality:	$x < 0$ or $2 < x \leq 10$
In interval notation:	$(-\infty, 0) \cup (2, 10]$

* OR (union): $\cup$, everything from the two sets are included
* AND (Intersection): $\cap$, what they have in common.

As you have seen, there are many ways of representing the same interval of values. These ways may include word descriptions or mathematical symbols.

The following statements and symbols ALL represent the same interval:	
WORDS:	**SYMBOLS:**
"all numbers between positive one and positive five, including the one and the five."	$1 \leq x \leq 5$
"x is less than or equal to 5 and greater than or equal to 1"	$\{x \in \mathbb{R} \mid 1 \leq x \leq 5\}$
"x is between 1 and 5, inclusive"	$[1,5]$

Name: ______________________________ Date: ________________

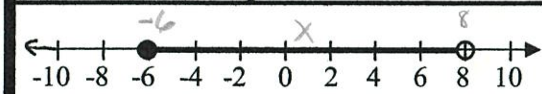
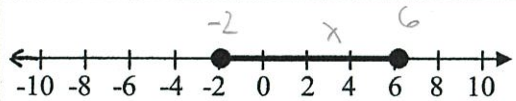
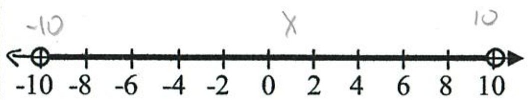
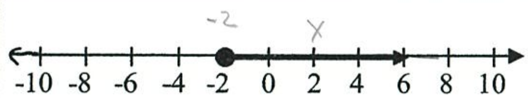
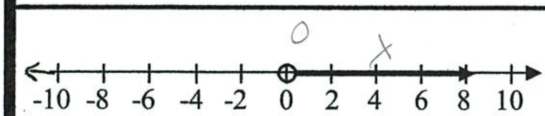
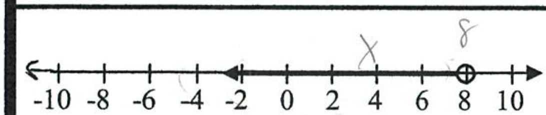
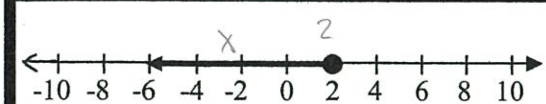
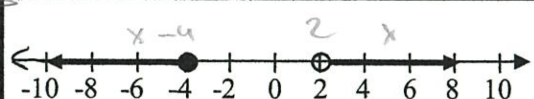
Interval Notation and Infinite Sets
Algebra 1

Sets of numbers that comprise **intervals** along a number line are of particular interest in mathematics. We have seen how to represent these intervals using **set builder notation**. Now we will introduce an alternative called **interval notation**. In this notation, [ ] are used for closed circles and ( ) are used for open circles and the number line is omitted. The interval $-3 < x \le 2$ would be written as $(-3, 2]$.

***Exercise* #1:** Sets representing intervals are shown on the number lines below. Represent each set using set builder notation and interval notation.

Numerical order

$\mathbb{R}$ = real #'s

Graphed Interval	Set Builder Notation	Interval Notation
	$\{x \in \mathbb{R} \mid -6 \le x < 8\}$	$[-6, 8)$
	$\{x \in \mathbb{R} \mid -2 \le x \le 6\}$	$[-2, 6]$
	$\{x \in \mathbb{R} \mid -10 < x < 10\}$	$(-10, 10)$
	$\{x \in \mathbb{R} \mid x \ge -2\}$	$[-2, \infty)$
	$\{x \in \mathbb{R} \mid x > 0\}$	$(0, \infty)$
	$\{x \in \mathbb{R} \mid x < 8\}$	$(-\infty, 8)$
	$\{x \in \mathbb{R} \mid x \le 2\}$	$(-\infty, 2]$
 OR	$\{x \in \mathbb{R} \mid x \le -4 \text{ OR } x > 2\}$	$(-\infty, -4] \cup (2, \infty)$

AND

Single

OR

Interval notation can be somewhat confusing because it closely resembles the way we specify a coordinate point (x, y). It will **always** be clear from the **context** of the problem whether you are dealing with a coordinate point or an interval. Thus, always read questions carefully to understand what is being asked.

***Exercise* #2:** The set given in set builder notation as $\{x : -3 < x \leq 7\}$ can also be expressed as which of the following?

( ]

(1) $(-3, 7)$ (3) $[-3, 7]$

(2) $(-3, 7]$ (4) $[-3, 7)$

2

$\frac{4}{2} > \frac{8}{2}$

$2 > 4$ ✓

$\frac{4}{-2} > \frac{8}{-2}$

$-2 \not> -4$

$-2 < -4$

***Exercise* #3:** The solution set to the inequality $-2x + 5 \geq 15$ can be expressed as

$-5 \quad -5$

$\frac{-2x}{-2} \geq \frac{10}{-2}$

switch direction of the inequality symbol when dividing by a negative

$x \leq -5$

(1) $(-\infty, -5]$ (3) $[-5, \infty)$

(2) $(-\infty, -5)$ (4) $(-5, \infty)$

-5

1

There is some additional terminology associated with intervals along a number line. If an interval contains all of its endpoints we call it **inclusive** and **closed**. If an interval lacks both of its endpoints we call it **exclusive** and **open**. If an interval contains one of its endpoints but not the other, we call it **half-closed (or half-open)**.

***Exercise* #4:** Which of the following inequalities represents the set of all real numbers between -8 and 4 inclusive?

[ ]

→ including

(1) $[-8, 4)$ (3) $(-8, 4)$

(2) $(-8, 4]$ (4) $[-8, 4]$

4

***Exercise* #5:** Which of the following intervals is half-closed?

(half-open)

1 of each

(1) $(5, \infty)$ (3) $[-6, 10]$

(2) $[3, 9)$ (4) $(-\infty, 2)$

2