

SOLVING INEQUALITIES

*An open sentence that contains $<$, $>$, \leq , or \geq is an **inequality**.

For all numbers a,b,c

Addition Property of Inequalities

If $a < b$, Then $a + c < b + c$
 If $a > b$, Then $a + c > b + c$

Subtraction Properties of Inequalities

If $a < b$, Then $a - c < b - c$
 If $a > b$, Then $a - c > b - c$

*These inequality properties simply mean that you are able to **add or subtract** the same number to each side of the inequality and you will **still get a true inequality**

Multiplication Property of Inequalities

If $a > b$ and $c > 0$, then $ac > bc$
 If $a < b$ and $c > 0$, then $ac < bc$
 If $a > b$ and $c < 0$, then $ac < bc$
 If $a < b$ and $c < 0$, then $ac > bc$

Division Property of Inequalities

If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$
 If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$
 If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$
 If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$

Handwritten examples of multiplication and division properties:

$$\begin{array}{l} 8 > -4 \\ \hline 2 & 2 \\ \hline 4 > -2 \end{array}$$

$$\begin{array}{l} 8 > 4 \\ \hline 2 & 2 \\ \hline 4 > 2 \end{array}$$

$$\begin{array}{l} 8 > 4 \\ \hline -2 & -2 \\ \hline -4 < -2 \end{array}$$

Must Switch

*If you **multiply** (or **divide**) each side of an inequality by a **positive** number, the **order** of the inequality remains the same.

****If you **multiply** (or **divide**) each side of an inequality by a **negative** number, the **order** of the inequality is **reversed**.

(Handwritten yellow box with asterisks and arrows pointing up from the bottom)

*An open sentence that contains $<$, $>$, \leq , or \geq is an **inequality**.

$<$	$>$	\leq	\geq
Less than	Greater than	Less than or equal to	Greater than or equal to

**If the same number is added (or subtracted) to each side of a true inequality, the resulting inequality is also true.

**You solve an inequality the same order you solve an equation (DCMS)

** If you multiply (or divide) each side of a true inequality by a positive number, then the inequality remains true, BUT if you multiply or divide by a **negative number**, you **MUST reverse** the directions of the inequality symbol to make it true!

** Answers may also be written in **set-builder notation**. Ex: $x \geq 20$, may also be written as $\{x | x \geq 20\}$ which is read the set of all numbers x such that x is greater than or equal to 20 or **interval notation** $[20, \infty)$

**You must graph your solution set!

The following table will show you how to shade your circle depending on the given inequality. Remember to shade in the direction that will match your final answer.

$<$	$>$	\leq	\geq
\circ	\circ	\bullet	\bullet

()

[]

Solving inequalities, like solving equations, consists of finding all values of the variable that make the inequality true.

Calc: 2nd Math to get symbol.

Exercise #1: Consider the linear inequality $2x + 3 > 11$.

(a) Circle each of the following values of x that lie in the solution set of this inequality.

~~$x = -2$~~

~~$x = \pi$~~

~~$x = 4$~~

$x = 4.1$

$x = 20$

(b) Solve the linear equation $2x + 3 = 11$.

$$\begin{array}{r} -3 \quad -3 \\ \hline 2x = 8 \\ \frac{2}{2} \quad \frac{8}{2} \\ \hline \boxed{x = 4} \end{array}$$

one solution

(c) What is the solution set of the linear inequality $2x + 3 > 11$?

$$\begin{array}{r} -3 \quad -3 \\ \hline 2x > 8 \\ \frac{2}{2} \quad \frac{8}{2} \\ \hline \boxed{x > 4} \end{array}$$

SB: $\{x \in \mathbb{R} \mid x > 4\}$
 many answers IN: $(4, \infty)$

Exercise #2: Which of the following represents the solution set of the inequality $11 \geq 4x + 3$?

(1) $x \leq 2$

(3) $x \geq 2$

(2) $x \leq -2$

(4) $x \geq -2$

$$\begin{array}{r} -3 \quad -3 \\ \hline 8 \geq 4x \\ \frac{8}{4} \geq \frac{4x}{4} \\ \hline \boxed{2 \geq x} \end{array}$$

OR $x \leq 2$

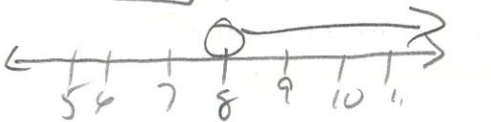
Exercise #3: Solve the following inequalities, and graph your solution. Also, list the properties you used and write your answer in set-builder and interval notation.

(a) $2x - 4 > 12$

$+4 \quad +4$ Add. Prop. of Ineq.

$\frac{2x}{2} > \frac{16}{2}$ Div. Prop. of Ineq.

$\boxed{x > 8}$



SB: $\{x \in \mathbb{R} \mid x > 8\}$

IN: $(8, \infty)$

(b) $12x - 14 \leq 9x + 13$

$-9x \quad -9x$ Sub. Prop. of Ineq.

$3x - 14 \leq 13$
 $+14 \quad +14$ Add. Prop. of Ineq.

$\frac{3x}{3} \leq \frac{27}{3}$ Div. Prop. of Ineq.

$\boxed{x \leq 9}$



SB: $\{x \in \mathbb{R} \mid x \leq 9\}$

IN: $(-\infty, 9]$

Exercise #4: Solve the following inequalities. Write your answer in set-builder and interval notation.

(a) $-\frac{1}{2}x + 5 \geq -7$

$-\frac{1}{2}x \geq -12$
 $x \leq 24$

A switch the direction of the symbol by a neg # multiplied by a neg #

SB: $\{x \in \mathbb{R} \mid x \leq 24\}$
 IN: $(-\infty, 24]$

(b) $-7x + 3 < 31$

$-7x < 28$
 $x > -4$

Switch direction by a neg #

Den. Switch or $7x < -28$ dividing by a neg #

$x > -4$

SB: $\{x \in \mathbb{R} \mid x > -4\}$
 IN: $(-4, \infty)$

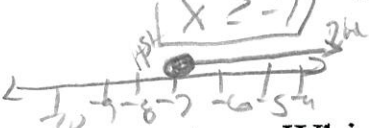
Exercise #5: Solve the following inequalities and graph your solution. Also, list the properties you used and write your answer in set-builder and interval notation.

(a) $5(2x + 11) \geq 3(x + 2)$

$10x + 55 \geq 3x + 6$
 $7x + 55 \geq 6$

Dist. Property
 Sub. Prop. of Ineq.
 Sub. Prop. of Ineq.

$7x \geq -49$
 $x \geq -7$



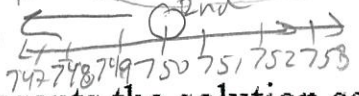
DN Prop. of Ineq.
 SB: $\{x \in \mathbb{R} \mid x \geq -7\}$
 IN: $[-7, \infty)$

(b) $0.10x + 40 > 0.12x + 25$

$40 > 0.02x + 25$
 $15 > 0.02x$

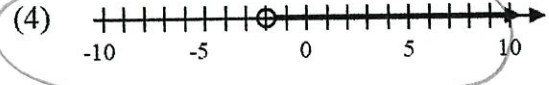
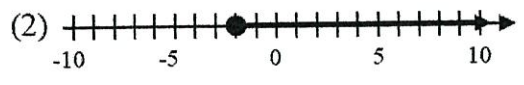
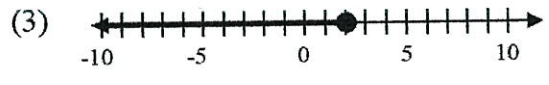
Sub. Prop. of Ineq.
 Sub. Prop. of Ineq.
 DN Prop. of Ineq.

$750 > x$
 or
 $x < 750$



SB: $\{x \in \mathbb{R} \mid x < 750\}$
 IN: $(-\infty, 750)$

Exercise #6: Which of the following represents the solution set of $16 > -3x + 10$?



$6 > -3x$
 $-2 < x$

switch direction when dividing by a neg #

Exercise #7: Translate each verbal sentence into an algebraic inequality.

(a) Eight is less than four times a number added to ten.

$8 < 4x + 10$

(b) Twice the sum of x and 1 is greater than or equal to 18.

$2(x + 1) \geq 18$

biconical