

Compound Inequalities

I. When two simple inequalities are combined into one statement by the words **AND** or **OR**, the result is called a **compound inequality**.

\cap \cup

WORDS	ALGEBRA	GRAPH
All real numbers less than 2 OR greater than 6	$x < 2$ OR $x > 6$ $x < 2 \cup x > 6$	
All real numbers less than or equal to 2 OR greater than or equal to 6	$x \leq 2$ OR $x \geq 6$ $x \leq 2 \cup x \geq 6$ $IN: (-\infty, 2] \cup [6, \infty)$	
All real numbers greater than 2 AND less than 6	$x > 2$ AND $x < 6$ $x > 2 \cap x < 6$ $IN: (2, 6)$ $2 < x < 6$	
All real numbers greater than or equal to 2 AND less than or equal to 6	$x \geq 2$ AND $x \leq 6$ $x \geq 2 \cap x \leq 6$ $IN: [2, 6]$ $2 \leq x \leq 6$	

"OR"
 2 inequalities are NOT connected
 "AND"
 2 inequalities are connected

Solving Inequalities Containing AND

A) When considered together, two inequalities such as $x < 10$ and $x > 0$ form a compound inequality. A compound inequality containing **and** is only true if both inequalities are true. *→ what they have in common*

In this diagram, oval A represents some integer solutions of $x < 10$, and oval B represents some integer solutions of $x > 0$. The overlapping region represents numbers that belong in both ovals. Those numbers are solutions of both $x < 10$ and $x > 0$.

You can graph the solutions of a compound inequality involving **AND** by using the idea of an overlapping region. The overlapping region is called the **intersection** and shows the numbers that are solutions of both inequalities.

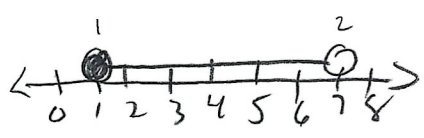
The statement $0 < x < 10$ can be read as x is greater than 0 and less than 10 or x is between 0 and 10

not including ()

$$\left. \begin{array}{l} -2 \leq x - 3 \\ +3 \quad +3 \end{array} \right\} \left. \begin{array}{l} x - 3 < 4 \\ +3 \quad +3 \end{array} \right\}$$

B) Examples: Solve and graph the intersection. Write your answer in set builder & interval notation.

1) $-2 \leq x - 3 < 4$
 $+3 \quad +3 \quad +3$
 $1 \leq x < 7$ AND

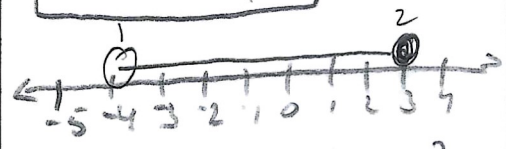


SB: $\{x \in \mathbb{R} \mid 1 \leq x < 7\}$

IN: $[1, 7)$

2) $-5 < 2x + 3 \leq 9$
 $-3 \quad -3 \quad -3$
 $-\frac{8}{2} < \frac{2x}{2} \leq \frac{6}{2}$

$-4 < x \leq 3$



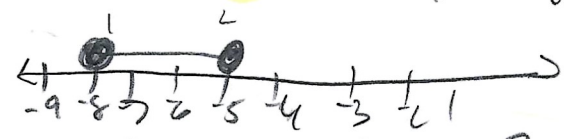
SB: $\{x \in \mathbb{R} \mid -4 < x \leq 3\}$

IN: $(-4, 3]$

*3) $y - 3 \geq -11$ and $y - 3 \leq -8$
 $+3 \quad +3 \quad +3 \quad +3$
 $y \geq -8$ and $y \leq -5$

$-8 \leq y \leq -5$

→ must be written as a single compound inequality



SB: $\{y \in \mathbb{R} \mid -8 \leq y \leq -5\}$

IN: $[-8, -5]$

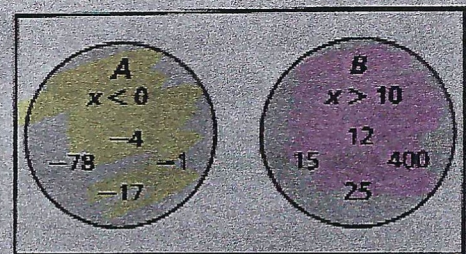
★ make sure to use the letter 'y'

★ And must be combined into 1 inequality ★

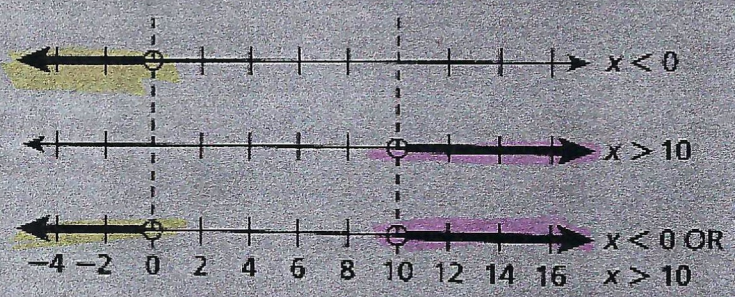
II. Solving Compound Inequalities Containing OR

A) Another type of compound inequality contains the word or. A compound inequality containing or is true if at least one of the inequalities is true. ★ only has to satisfy one

In this diagram, circle A represents some integer solutions of $x < 0$, and circle B represents some integer solutions of $x > 10$. The combined shaded regions represent numbers that are solutions of either $x < 0$ or $x > 10$.



You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combined regions are called the union and show the numbers that are solutions of either inequality.



all together"

**When solving problems involving inequalities, within is meant to be inclusive, so use \leq or \geq . Between is meant to be exclusive, so use $<$ or $>$.

Examples: Solve and graph the union. Write your answer in set builder & interval notation.

1) $\frac{2x}{2} \leq \frac{6}{2}$ or $\frac{3x}{3} > \frac{12}{3}$

$x \leq 3$ OR $x > 4$

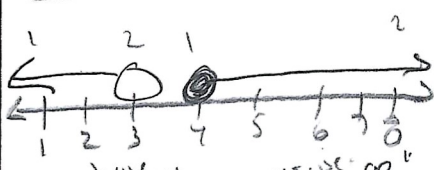


SB: $\{x \in \mathbb{R} \mid x \leq 3 \text{ or } x > 4\}$

IN: $(-\infty, 3] \cup (4, \infty)$

2) $a + 1 < 4$ U $a - 1 \geq 3$

$a < 3$ U $a \geq 4$



SB: $\{a \in \mathbb{R} \mid a < 3 \text{ or } a \geq 4\}$

IN: $(-\infty, 3) \cup [4, \infty)$

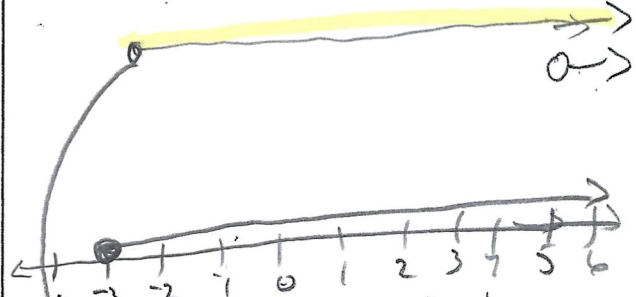
You do NOT combine OR into a single compound inequality

SB + IN
MUST
recheck
graph

*3) $-2m + 7 \leq 13$ or $5m + 12 > 37$

$-2m \leq 6$ or $5m > 25$
 $m \geq -3$ or $m > 5$

$m \geq -3$ OR $m > 5$



pick the one that covers more (bigger)

SB: $\{m \in \mathbb{R} \mid m \geq -3\}$
IN: $[-3, \infty)$

***What is the solution set of the inequality $-7 < x + 2 < 4$

a) $\{x \mid -5 < x < 6\}$

b) $\{x \mid -5 < x < 2\}$

c) $\{x \mid -9 < x < 2\}$

d) $\{x \mid -9 < x < 6\}$

$-9 < x < 2$