PS: {0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225}

Name: Key Date: ____________

Factoring the Difference of Perfect Squares
Algebra 1

DOPS

In algebra, there are three very common types of factoring. In the last lesson, you learned how to factor out the GCF from an expression. Today we will work with a pattern of factoring called the Difference of Perfect Squares. First, we develop the idea of a perfect square.

***Exercise* #1:** Write each of the following expressions as a monomial squared.

(a) $49 =$ $(7)^2$

(b) $16x^2 =$ $(4x)^2$

(c) $64y^2 =$ $(8y)^2$

(d) $100y^2 =$ $(10y)^2$

Each of the preceding monomials is considered a **perfect square** because it can be written as the square of another monomial. It is essential for you to be able to distinguish between monomials that are perfect squares and those that are not.

***Exercise* #2:** Classify each of the following as a perfect square or not a perfect square.

(a) $9x^2$ yes! $(3x)^2$

(b) $4x$ NO

(c) $8x^2$ NO.

(d) 25 yes $(5)^2$

Before we truly begin our factoring today, we will review multiplying binomials that are **conjugates** of one another. **Conjugate** binomials **only differ** in the **sign** that connects the two monomials.

***Exercise* #3:** Write each of the following products without the parentheses.

(a) $(x-2)(x+2)=$
$x^2+\cancel{2x}-\cancel{2x}-4$
$\boxed{x^2-4}$

(b) $(3x+1)(3x-1)=$
$9x^2-3x+3x-1$
$\boxed{9x^2-1}$

(c) $(4y-5)(4y+5)=$
$16y^2+20y-20y-25$
$\boxed{16y^2-25}$

DOPS: Requirements: ① must be a binomial ② must be subtraction ③ Terms must be perfect squares

Notice that when multiplying conjugates the inner and outer terms cancel and we are left with only the difference of perfect squares. We can reverse this process if we recognize that we have a binomial that is the difference of two perfect squares. ★ $a^2-b^2=(a+b)(a-b)$

***Exercise* #4:** Write each of the following binomials as the product of two conjugate pairs. In other words, factor the following binomials.

(a) x^2-16 $\boxed{(x+4)(x-4)}$

(b) x^2-81 $\boxed{(x+9)(x-9)}$

(c) y^2-1 $\boxed{(y+1)(y-1)}$

Creating Conjugates

steps: ① write 2 sets of ( ). one w/a + and one w/a −.
② Take the √ of 1st term & place the answer in 1st spot of each ( )
③ Take the √ of 2nd term + place the answer in 2nd spot of each ( ).

(g) $0.16y^4 - 0.09$ | (h) $p^2 - .36$ | (i) $x^2y^2 - 100$ | (j) $144a^2b^2 - 225x^4y^2$

$\boxed{(0.4y^2 - 0.3)(0.4y^2 + 0.3)}$ $\boxed{(p-0.6)(p+0.6)}$ $\boxed{(xy+10)(xy-10)}$ $\boxed{(12ab + 15x^2y)(12ab - 15x^2y)}$

***Exercise* #5:** Factor each of the following binomials.

(a) $4x^2 - 9$

$\boxed{(2x+3)(2x-3)}$

(b) $25x^2 - 1$

$\boxed{(5x+1)(5x-1)}$

(c) $9x^2 - 49y^2$

$\boxed{(3x+7y)(3x-7y)}$

(d) $121 - y^2$

$\boxed{(11+y)(11-y)}$

(e) $x^2 - \frac{9}{16}$

$\boxed{(x+\frac{3}{4})(x-\frac{3}{4})}$

(f) $9 - \frac{25}{4}x^2$

$\boxed{(3-\frac{5}{2}x)(3+\frac{5}{2}x)}$

We should now feel comfortable with factoring out GCF's from expressions and factoring binomials that have the form of the difference of perfect squares. We can combine these two into a process known as **complete factoring**. Complete factoring is factoring an expression until it cannot be factored any further. → 2 steps (2 types of factoring)

***Exercise* #6:** Consider the binomial $5x^2 - 20$.

(a) Is this binomial in the form of the difference of perfect squares? Explain. NO! B/C 5 & 20 are not P.S.'s

(b) Write this binomial as the product of its GCF and a binomial.

$5(x^2-4)$ Step 1: GCMF

(c) Can anything else be factored in the expression you wrote in (b)? If so, write the expression in its **completely factored form.**

$\boxed{5(x+2)(x-2)}$ step 2: DOPS.

***Exercise* #7:** Completely factor each of the following binomials.

(a) $4x^2 - 36$

$4(x^2-9)$ GCMF

$\boxed{4(x+3)(x-3)}$ DOPS

(b) $2x^2 - 2y^2$

$2(x^2-y^2)$ GCMF

$\boxed{2(x+y)(x-y)}$ DOPS

(c) $20 - 45x^2$

$5(4-9x^2)$ GCMF

$\boxed{5(2-3x)(2+3x)}$ DOPS

(d) $3x^3 - 27x$

$3x(x^2-9)$ GCMF

$\boxed{3x(x+3)(x-3)}$ DOPS

(e) $24x - 6x^3$

$6x(4-x^2)$ GCMF

$\boxed{6x(2-x)(2+x)}$ DOPS

(f) $10x^3 - 250x$

$10x(x^2-25)$ GCMF

$\boxed{10x(x+5)(x-5)}$ DOPS

★ If the area of a square is $4x^2 - 25$ how can you write this as the product of 2 binomials?

$A = S^2$ $\boxed{(2x-5)(2x+5)}$