Name: Key

8A: Algebra 1

Date:__________

Period_______

Simplifying Radicals Day I

Here are some key definitions.

- **Root:** One of two or more equal factors of a number.
- **Square root:** One of two equal factors of a number.
- **Radical sign** $\sqrt{}$: Symbol meaning *the square root of*. For example: If $\sqrt{a} = x$, then $x \bullet x = a$ or $x^2 = a$.
- **Principal square root**: Every positive number has two square roots. The principal square root is positive. The other root is negative. For example: $3 \bullet 3 = 9$ and $-3 \bullet (-3) = 9$, so $\sqrt{9} = 3$ (principal square root) and -3
- **Radicand**: The number under the radical sign. In $\sqrt{10}$, for example, 10 is the radicand.
- **Coefficient**: A number by which a square root is multiplied. In $3\sqrt{10}$, for example, 3 is the coefficient. The meaning is *three times the square root of ten.*
- **Perfect square**: A number that has a rational square root. For example:

$\sqrt{9} = 3$ $\sqrt{25} = 5$ $\sqrt{\frac{9}{25}} = \frac{3}{5}$

A negative number does not have a real-number square root.

ex $\sqrt{-25}$, $\sqrt{-16}$

NOT Real, imaginary

$\frac{\#}{0}$ = undefined

The multiplication of square roots is an important skill to develop for a variety of applications.

THE MULTIPLICATION PROPERTY OF SQUARE ROOTS

$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$ for all real numbers, a and b, such that $a \geq 0$ and $b \geq 0$

Exercise #1: Verify the Multiplication Property of Square Roots for $\sqrt{4}$ and $\sqrt{9}$.

$a=4$ $b=9$

$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$

$\sqrt{4} \cdot \sqrt{9} = \sqrt{4 \cdot 9}$

$2 \cdot 3 = \sqrt{36}$

$6 = 6$ ✓

Exercise #2: Evaluate each of the following square root products.

(a) $\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8}$

$a=2$

$b=8$

$= \sqrt{16}$

$= \boxed{4}$

(b) $\sqrt{3} \cdot \sqrt{12} = \sqrt{3 \cdot 12}$

$a=3$

$b=12$

$= \sqrt{36}$

$= \boxed{6}$

(c) $\sqrt{5} \cdot \sqrt{20} = \sqrt{5 \cdot 20}$

$a=5$

$b=20$

$= \sqrt{100}$

$= 10$

Simplifying Irrational Square Roots – Many times we want to write an irrational number in its "simplest" form by taking the square root of all perfect squares that are factors of the number. We do this by reversing the Multiplication Property.

Simplifying Square Roots

Steps: A radical is **simplified** when the radicand is the smallest possible positive integer. To simplify a square root:

① • Factor the radicand so that one factor is a perfect square. For example: (*choose the biggest P.S. if there is more than 1.)

$\sqrt{27} = \sqrt{9 \bullet 3}$ (9 is a perfect square)

② • Simplify this factor and write its square root as a coefficient:

$\sqrt{27} = \sqrt{9 \bullet 3} = 3\sqrt{3}$ (the square root of 9 is 3, which becomes the coefficient)

Remember:

$\sqrt{n} \bullet \sqrt{n} = n$

$(\sqrt{n})^2 = n$

$\sqrt{n^2} = n$

$\sqrt{5}\cdot\sqrt{5} = \sqrt{5\cdot5} = \sqrt{25} = 5$

$(\sqrt{3})^2 = \sqrt{3}\cdot\sqrt{3} = 3$

$\sqrt{8}\cdot\sqrt{8} = 8$

P.S.: {0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225...}

$(\sqrt{16})^2 = 16$

$(\sqrt{7})^2 = 7$

$\sqrt{9^2} = \sqrt{9\cdot9} = \sqrt{81} = 9$

Exercise #1: Express each of the following square roots in simplest radical form. (↳ No decimal)

(a) $\sqrt{8}$

$\sqrt{4}\cdot\sqrt{2}$

$\boxed{2\sqrt{2}}$

(b) $\sqrt{18}$

$\sqrt{9}\cdot\sqrt{2}$

$\boxed{3\sqrt{2}}$

(c) $\sqrt{28}$

$\sqrt{4}\cdot\sqrt{7}$

$\boxed{2\sqrt{7}}$

(d) $\sqrt{45}$

$\sqrt{9}\cdot\sqrt{5}$

$\boxed{3\sqrt{5}}$

(e) $4\sqrt{27}$

$4\cdot\sqrt{9}\cdot\sqrt{3}$

$4\cdot3\sqrt{3}$

$\boxed{12\sqrt{3}}$

(f) $-3\sqrt{20}$

$-3\cdot\sqrt{4}\cdot\sqrt{5}$

$-3\cdot2\sqrt{5}$

$\boxed{-6\sqrt{5}}$

(g) $5\sqrt{48}$

$5\cdot\sqrt{16}\cdot\sqrt{3}$

$5\cdot4\cdot\sqrt{3}$

$\boxed{20\sqrt{3}}$

$5\sqrt{48}$

$5\cdot\sqrt{4}\cdot\sqrt{12}$

$5\cdot2\sqrt{12} = 10\sqrt{12}$

$10\cdot\sqrt{4}\sqrt{3}$

$10\cdot2\sqrt{3}$

$20\sqrt{3}$

(h) $-\frac{2}{5}\sqrt{75}$

$-\frac{2}{5}\cdot\sqrt{25}\cdot\sqrt{3}$

$-\frac{2}{5}\cdot5\sqrt{3}$

$\boxed{-2\sqrt{3}}$

(i) $\frac{\sqrt{24}}{2}$

Num: $\sqrt{24}$ Denom: 2

$\sqrt{4}\sqrt{6}$

$2\sqrt{6}$

$\frac{2\sqrt{6}}{2} = \sqrt{6}$

OR $\frac{1}{2}\sqrt{24}$

$\frac{1}{2}\cdot\sqrt{4}\cdot\sqrt{6}$

$\frac{1}{2}\cdot2\sqrt{6}$

$\boxed{1\sqrt{6}} = \boxed{\sqrt{6}}$

(j) The principal square of 64 is: (positive)

$\boxed{8}$

(k) Simplify: $\sqrt{162}$

$\sqrt{81}\cdot\sqrt{2}$

$\boxed{9\sqrt{2}}$

(l) Simplify $\sqrt{200}$

$\sqrt{100}\cdot\sqrt{2}$

$\boxed{10\sqrt{2}}$

(m) Simplify: $\sqrt{864}$

$\sqrt{144}\cdot\sqrt{6}$

$\boxed{12\sqrt{6}}$

II. Express each of the following square roots in simplest radical form.

1) $\sqrt{108}$ $\sqrt{36} \cdot \sqrt{3}$ $\boxed{6\sqrt{3}}$	2) $\sqrt{12}$ $\sqrt{4} \cdot \sqrt{3}$ $\boxed{2\sqrt{3}}$
3) $\sqrt{20}$ $\sqrt{4 \cdot 5}$ $\boxed{2\sqrt{5}}$	4) $\sqrt{128}$ $\sqrt{64} \cdot \sqrt{2}$ $\boxed{8\sqrt{2}}$
5) $\sqrt{300}$ $\sqrt{100} \cdot \sqrt{3}$ $\boxed{10\sqrt{3}}$	6) $3\sqrt{250}$ $3 \cdot \sqrt{25} \cdot \sqrt{10}$ $3 \cdot 5\sqrt{10}$ $\boxed{15\sqrt{10}}$
7) $-5\sqrt{243}$ $-5 \cdot \sqrt{81} \cdot \sqrt{3}$ $-5 \cdot 9\sqrt{3}$ $\boxed{-45\sqrt{3}}$	8) $\frac{1}{2}\sqrt{80}$ $\frac{1}{2} \cdot \sqrt{16} \cdot \sqrt{5}$ $\frac{1}{2} \cdot 4\sqrt{5}$ $\boxed{2\sqrt{5}}$
9) $-\sqrt{125}$ $-1 \cdot \sqrt{25} \cdot \sqrt{5}$ $-1 \cdot 5\sqrt{5}$ $\boxed{-5\sqrt{5}}$	10) $\frac{1}{4}\sqrt{96}$ $\frac{1}{4} \cdot \sqrt{16} \cdot \sqrt{6}$ $\frac{1}{4} \cdot 4\sqrt{6}$ $\boxed{1\sqrt{6}}$