

### How Do We Solve Quadratic Equations?

I. An equation of the **second degree** (highest exponent of 2) is called a

Quadratic equation. Ex:  $x^2 + 4x + 4 = 0$

II. The **standard form** of a quadratic equation in one variable is:  $ax^2 + bx + c = 0$   
where a, b, c are real numbers and  $a \neq 0$ . (Must be in DPO and = to 0).

\*\*Also, there can be no  $-x^2$  (we don't know how to factor a  $-x^2$  polynomial)

\*\*Quadratic equations must always be in standard form before you solve them!

Write the following equation in standard form:  $-3x - 10 = -x^2$

$$\begin{array}{r} +x^2 \quad \quad +x^2 \\ \hline x^2 - 3x - 10 = 0 \end{array}$$

III. Examples:

Transform the following equations into an equivalent quadratic equation in standard form.

$$\begin{array}{r} 1) 5x^2 - 3x = 15 \\ - \quad -15 \quad -15 \\ \hline 5x^2 - 3x - 15 = 0 \end{array}$$

$$\begin{array}{r} 2) 10g + 8 = g^2 \\ -10g - 8 \quad -10g - 8 \\ \hline 0 = g^2 - 10g - 8 \\ \hline g^2 - 10g - 8 = 0 \end{array}$$

$$\begin{array}{r} 3) w(w - 4) = 5 \\ w^2 - 4w = 5 \\ -5 \quad -5 \\ \hline w^2 - 4w - 5 = 0 \end{array}$$

IV. The following property is very important in solving equations:

#### Zero Product Property

If a and b are real numbers, then:  
 **$ab = 0$  if and only if  $a = 0$  or  $b = 0$**

$$\begin{array}{l} 5 \cdot 0 = 0 \\ 0 \cdot 5 = 0 \\ 0 \cdot 0 = 0 \end{array}$$

This principle is used to solve quadratic equations. For example, to solve the quadratic equation  $x^2 - 3x + 2 = 0$ , we can write it as  $(x - 2)(x - 1) = 0$ . The factors  $(x - 2)$  and  $(x - 1)$  represent real numbers whose product is 0. The equation will be true if either one of the factors is 0, that is, if  $(x - 2) = 0$  or if  $(x - 1) = 0$ .

Steps

- Example: Solve the following quadratic equation:  
 Step 1: Make sure the equation is in standard form  
 Step 2: Factor the equation

$x^2 - 3x + 2 = 0$  *2 answers*

Step 3: Set both factors equal to 0 (The "T")

$$(x-2)(x-1) = 0$$

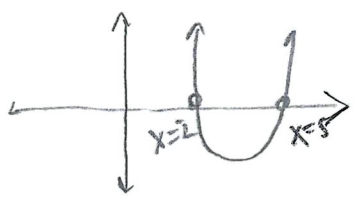
$x-2=0$	$x-1=0$
$+2 \quad +2$	$+1 \quad +1$
$x=2$	$x=1$

Step 4: Solve both sides of the "T"

Step 5: Check each answer in original equation

Check for  $x=2$ :

Check for  $x=1$ :



$$x^2 - 3x + 2 = 0$$

$$(2)^2 - 3(2) + 2 = 0$$

$$4 - 6 + 2 = 0$$

$$0 = 0$$

$$x^2 - 3x + 2 = 0$$

$$(1)^2 - 3(1) + 2 = 0$$

$$1 - 3 + 2 = 0$$

$$0 = 0$$

Solve the following Quadratic Equations:

1)  $x^2 - 7x = -10$  *2 answers*

$$x^2 - 7x + 10 = 0$$

$$(x-5)(x-2) = 0$$

$x-5=0$	$x-2=0$
$+5 \quad +5$	$+2 \quad +2$
$x=5$	$x=2$

Check #1  $x=5$

$$x^2 - 7x = -10$$

$$(5)^2 - 7(5) = -10$$

$$25 - 35 = -10$$

$$-10 = -10$$

Check #2  $x=2$

$$x^2 - 7x = -10$$

$$(2)^2 - 7(2) = -10$$

$$4 - 14 = -10$$

$$-10 = -10$$

*check into the originals always!*

2)  $2x^2 = 3x$

$$2x^2 - 3x = 0$$

$$x(2x-3) = 0$$

$x=0$	$2x-3=0$
	$+3 \quad +3$
	$\frac{2x}{2} = \frac{3}{2}$
	$x = 1\frac{1}{2}$

$\{2, 5\}$   
 $x=2, 5$   
 • Roots  
 • solutions  
 • zeros

3)  $4z^2 - 36 = 0$

$$4(z^2 - 9) = 0$$

$$4(z-3)(z+3) = 0$$

$z-3=0$	$z+3=0$
$+3 \quad +3$	$-3 \quad -3$
$z=3$	$z=-3$

$\{-3, 3\}$

4)  $5x^2 - 20 = 0$

$$5(x^2 - 4) = 0$$

$$5(x+2)(x-2) = 0$$

$x+2=0$	$x-2=0$
$-2 \quad -2$	$+2 \quad +2$
$x=-2$	$x=2$

$\{-2, 2\}$

$$5) 3x^2 = -5x + 2$$

$$+5x + 2 \quad +5x + 2$$

$$\textcircled{3}x^2 + 5x - 2 = 0$$

$$\left(\frac{3x}{3} + \frac{6}{3}\right)\left(\frac{3x}{3} - 1\right) = 0$$

$$(x + 2)(3x - 1) = 0$$

$$x + 2 = 0$$

$$\cancel{-2} \quad \cancel{-2}$$

$$\boxed{x = -2}$$

$$3x - 1 = 0$$

$$\cancel{+1} \quad \cancel{-1}$$

$$\frac{3x}{3} = \frac{1}{3}$$

$$\boxed{x = \frac{1}{3}}$$

$$\{-2, \frac{1}{3}\}$$

$$7) \frac{x+5}{3} = \frac{10}{x-8}$$

$$x \neq 8$$

$$(x+5)(x-8) = 30$$

$$x^2 - 8x + 5x - 40 = 30$$

$$x^2 - 3x - 40 = 30$$

$$\cancel{-30} \quad \cancel{-30}$$

$$x^2 - 3x - 70 = 0$$

$$(x - 10)(x + 7) = 0$$

$$x - 10 = 0$$

$$\cancel{+10} \quad \cancel{+10}$$

$$\boxed{x = 10}$$

$$x + 7 = 0$$

$$\cancel{-7} \quad \cancel{-7}$$

$$\boxed{x = -7}$$

$$\{-7, 10\}$$

$$9) \frac{x+2}{2} = \frac{12}{x}$$

$$x \neq 0$$

$$x(x+2) = 24$$

$$x^2 + 2x = 24$$

$$\cancel{-24} \quad \cancel{-24}$$

$$x^2 + 2x - 24 = 0$$

$$(x + 6)(x - 4) = 0$$

$$x + 6 = 0$$

$$\cancel{-6} \quad \cancel{-6}$$

$$\boxed{x = -6}$$

$$x - 4 = 0$$

$$\cancel{+4} \quad \cancel{+4}$$

$$\boxed{x = 4}$$

$$\{-6, 4\}$$

$$6) \frac{x-2}{2} = \frac{2}{x}$$

$$x \neq 0$$

$$x^2 = 4$$

$$\cancel{-4} \quad \cancel{-4}$$

$$x^2 - 4 = 0$$

$$(x - 2)(x + 2) = 0$$

$$x - 2 = 0$$

$$x + 2 = 0$$

$$\cancel{+2} \quad \cancel{-2}$$

$$\boxed{x = 2}$$

$$\boxed{x = -2}$$

$$\{-2, 2\}$$

$$*8) \frac{x-2}{x} = \frac{5}{x}$$

$$x \neq 0$$

$$x(x-2) = 5x$$

$$x^2 - 2x = 5x$$

$$\cancel{-5x} \quad \cancel{-5x}$$

$$x^2 - 7x = 0$$

$$x(x - 7) = 0$$

$$\boxed{x = 0}$$

Reject!  
Can't divide  
by zero

$$x - 7 = 0$$

$$\cancel{+7} \quad \cancel{+7}$$

$$\boxed{x = 7}$$

$$\{7\}$$

$$10) x(x+3) = 40$$

$$x^2 + 3x = 40$$

$$\cancel{-40} \quad \cancel{-40}$$

$$x^2 + 3x - 40 = 0$$

$$(x + 8)(x - 5) = 0$$

$$x + 8 = 0$$

$$x - 5 = 0$$

$$\cancel{-8} \quad \cancel{-8}$$

$$\cancel{+5} \quad \cancel{+5}$$

$$\boxed{x = -8}$$

$$\boxed{x = 5}$$

$$\{-8, 5\}$$