

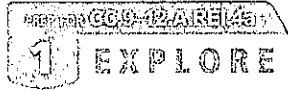
# Completing the Square

Getting Ready

**Essential question:** How can you solve quadratic equations without factoring?



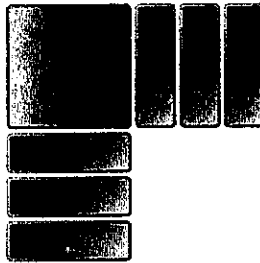
Video Tutor



## Completing the Square

The diagram below represents the expression  $x^2 + 6x + c$  with the constant term missing.

- A Complete the diagram by filling the bottom right corner with 1-tiles to form a square.



- B How many 1-tiles did you add to the expression? \_\_\_\_\_
- C Write the trinomial represented by the algebra tiles for the complete square.

$$x^2 + \quad x + \quad$$

- D You should recognize this trinomial as an example of the special case  $(a + b)^2 = a^2 + 2ab + b^2$ . Recall that trinomials of this form are called perfect square trinomials. Since the trinomial is a perfect square, you can factor it into two binomials that are the same.

$$x^2 + \quad x + \quad = \left( \quad x + \quad \right)^2$$

### REFLECT

- 1a. Look at the algebra tiles above. The  $x$ -tiles are divided equally, with 3 tiles on the right and bottom sides of the  $x^2$ -tile. How does the number 3 relate to the total number of  $x$ -tiles? How does the number 3 relate to the number of 1-tiles you added?

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- 1b. How would algebra tiles be arranged to form a perfect square trinomial  $x^2 + 8x + c$ ? How many 1-tiles must be added? How is this number related to the number of  $x$ -tiles?

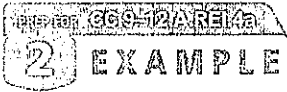
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**Completing the Square** Finding the value of  $c$  needed to make an expression such as  $x^2 + 6x + c$  into a perfect square trinomial is called **completing the square**.

Using algebra tiles, half of the  $x$ -tiles are placed along the right and bottom sides of the  $x^2$ -tile. The number of 1-tiles added is the square of the number of  $x$ -tiles on either side of the  $x^2$ -tile.

To complete the square for the expression  $x^2 + bx + c$ , replace  $c$  with  $\left(\frac{b}{2}\right)^2$ . The perfect square trinomial is  $x^2 + bx + \left(\frac{b}{2}\right)^2$  and factors as  $\left(x + \frac{b}{2}\right)^2$ .



**EXAMPLE** Completing the Square

Complete the square to form a perfect square trinomial. Then factor the trinomial.

A  $x^2 + 12x + c$

Identify  $b$ .  $b = \underline{\hspace{2cm}}$

Find  $c$ .  $c = \left(\frac{b}{2}\right)^2 = \left(\frac{\hspace{1cm}}{2}\right)^2 = \underline{\hspace{2cm}}$

Write the trinomial.  $x^2 + \hspace{1cm} x + \hspace{1cm}$

Factor the trinomial.  $x^2 + \hspace{1cm} x + \hspace{1cm} = \left(\hspace{1cm}\right)^2$

B  $z^2 - 24z + c$

Identify  $b$ .  $b = \underline{\hspace{2cm}}$

Find  $c$ .  $c = \left(\frac{b}{2}\right)^2 = \left(\frac{\hspace{1cm}}{2}\right)^2 = \underline{\hspace{2cm}}$

Write the trinomial.  $z^2 + \hspace{1cm} z + \hspace{1cm}$

Factor the trinomial.  $z^2 + \hspace{1cm} z + \hspace{1cm} = \left(\hspace{1cm}\right)^2$

**REFLECT**

2a. In Part A,  $b$  is positive and in Part B,  $b$  is negative. Does this affect the sign of  $c$ ? Why or why not?

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2b. How can you confirm that you have factored each trinomial correctly?

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## Completing The Square Continued

\*You can solve any quadratic equation by completing the square. This method turns every expression  $x^2 + bx$  into a perfect-square trinomial. You complete the square by adding  $\left(\frac{b}{2}\right)^2$  to  $x^2 + bx$ , where b is the coefficient of the x-term.

\* When a trinomial is a perfect square, there is a relationship between the coefficient of the x-term and the constant term.

$$x^2 + 6x + 9$$

$$\left(\frac{6}{2}\right)^2 = 9$$

$$x^2 - 8x + 16$$

$$\left(\frac{-8}{2}\right)^2 = 16$$

WORDS	NUMBERS	ALGEBRA
To complete the square of $x^2 + bx$ , add $\left(\frac{b}{2}\right)^2$ to the expression. This will form a perfect-square trinomial.	$x^2 + 6x + \underline{\hspace{2cm}}$ $x^2 + 6x + \left(\frac{6}{2}\right)^2$ $x^2 + 6x + 9$ $(x + 3)^2$	$x^2 + bx + \underline{\hspace{2cm}}$ $x^2 + bx + \left(\frac{b}{2}\right)^2$ $\left(x + \frac{b}{2}\right)^2$

Examples: Complete the square to form a perfect-square trinomial. Then factor the trinomial.

1)  $x^2 + 32x$

2)  $c^2 - 16c$

3)  $8x + x^2$

4)  $w^2 - 13w$

$$5) x^2 - 9x$$

$$6) x^2 + 22x$$

$$7) m^2 + 10m$$

$$8) x^2 - 5x$$

$$9) x^2 + 15x$$

$$10) y^2 + 2y$$

$$11) b^2 + 4b$$

$$12) g^2 - 20g + 15$$

$$13) x^2 + 17x + 25$$

$$14) x^2 - 8x - 18$$