Name Key Class Date

8-8

Completing the Square

Going Deeper

Essential question: *How can you solve quadratic equations without factoring?*

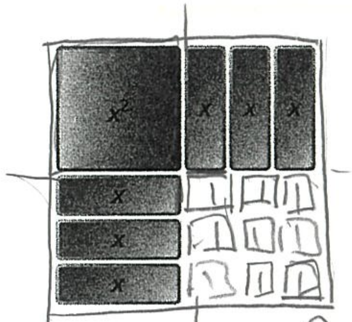
Video Tutor

PREP FOR CC.9-12.A.REI.4a

1 EXPLORE Completing the Square ax^2+bx+c

The diagram below represents the expression $x^2 + 6x + c$ with the constant term missing.

A Complete the diagram by filling the bottom right corner with 1-tiles to form a square.



B How many 1-tiles did you add to the expression? 9

C Write the trinomial represented by the algebra tiles for the complete square.

$x^2 +$ 6 $x +$ 9 $c=\left(\frac{b}{2}\right)^2$ $c=\left(\frac{6}{2}\right)^2$ $c=(3)^2$ $c=9$

D You should recognize this trinomial as an example of the special case $(a + b)^2 = a^2 + 2ab + b^2$. Recall that trinomials of this form are called perfect square trinomials. Since the trinomial is a perfect square, you can factor it into two binomials that are the same.

$x^2 +$ 6 $x +$ 9 $=$ $(x +$ 3 $)^2$ $\frac{b}{2}$

$(x+3)(x+3)$ $\frac{b}{2}$

REFLECT

1a. Look at the algebra tiles above. The x-tiles are divided equally, with 3 tiles on the right and bottom sides of the x^2-tile. How does the number 3 relate to the total number of x-tiles? How does the number 3 relate to the number of 1-tiles you added?

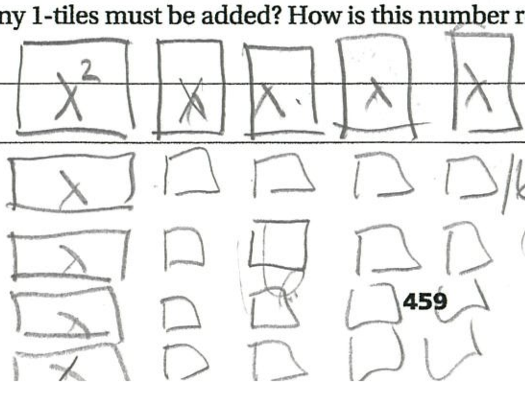
3 is half of 6. The constant 9 is 3^2

1b. How would algebra tiles be arranged to form a perfect square trinomial $x^2 + 8x + c$? How many 1-tiles must be added? How is this number related to the number of x-tiles?

$b=8$

$(x+4)^2$

$\frac{b}{2}$



16 1 tiles

The c-term is half of b

The # of x-tiles ÷ then squared

$c=\left(\frac{b}{2}\right)^2$

$c=\left(\frac{8}{2}\right)^2$

$c=(4)^2$

$c=16$

Completing the Square Finding the value of c needed to make an expression such as $x^2 + 6x + c$ into a perfect square trinomial is called **completing the square.**

Using algebra tiles, half of the x-tiles are placed along the right and bottom sides of the x^2-tile. The number of 1-tiles added is the square of the number of x-tiles on either side of the x^2-tile.

steps

To complete the square for the expression $x^2 + bx + c$, replace c with $\left(\frac{b}{2}\right)^2$. The perfect square trinomial is $x^2 + bx + \left(\frac{b}{2}\right)^2$ and factors as $\left(x + \frac{b}{2}\right)^2$.

PREP FOR CC.9-12.A.REI.4a

2 EXAMPLE Completing the Square

Complete the square to form a perfect square trinomial. Then factor the trinomial.

ax^2+bx+c

A $x^2 + 12x + c$

Identify b. $b =$ 12

Find c. $c = \left(\frac{b}{2}\right)^2 = \left(\frac{12}{2}\right)^2 = (6)^2 = 36$

Write the trinomial. $x^2 + 12x + 36$

Factor the trinomial. $x^2 + 12x + 36 = \boxed{(x+6)^2}$

$(x+6)(x+6)$

$\frac{b}{2}$

B $z^2 - 24z + c$

Identify b. $b =$ -24

Find c. $c = \left(\frac{b}{2}\right)^2 = \left(\frac{-24}{2}\right)^2 = (-12)^2 = 144$

Write the trinomial. $z^2 - 24z + 144$

Factor the trinomial. $z^2 - 24z + 144 = \boxed{(z-12)^2}$

$(z-12)(z-12)$

$\frac{b}{2}$

REFLECT

2a. In Part A, b is positive and in Part B, b is negative. Does this affect the sign of c? Why or why not?

The sign of b has No effect on the sign of c B/c $c = \left(\frac{b}{2}\right)^2$ and any non-zero # squared is always positive so "c" is always positive.

2b. How can you confirm that you have factored each trinomial correctly?

After you write the binomial twice, you Double Distribute to check.

Completing The Square Continued

You can solve any quadratic equation by completing the square. This method turns every expression $x^2 + bx$ into a perfect-square trinomial. You complete the square by adding $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$, where b is the coefficient of the x-term.

= C   b

* When a trinomial is a perfect square, there is a relationship between the coefficient of the x-term and the constant term.

C

$$x^2 + 6x + 9 \qquad x^2 - 8x + 16$$

$$\left(\frac{6}{2}\right)^2 = 9 \qquad \left(\frac{-8}{2}\right)^2 = 16$$

$$(x+3)^2 \;\; \frac{b}{2} \qquad (x-4)^2 \;\; \frac{b}{2}$$

WORDS	NUMBERS	ALGEBRA
To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$ to the expression. This will form a perfect-square trinomial.	$x^2 + 6x +$ ______ $x^2 + 6x + \left(\frac{6}{2}\right)^2$ $x^2 + 6x + 9$ $(x + 3)^2$	$x^2 + bx +$ _____ $x^2 + bx + \left(\frac{b}{2}\right)^2$ $\left(x + \frac{b}{2}\right)^2$

①   ②

Examples: Complete the square to form a perfect-square trinomial. Then factor the trinomial.

1) $x^2 + 32x + C$ $\boxed{b = 32}$

$x^2 + 32x + \left(\frac{b}{2}\right)^2$

$x^2 + 32x + \left(\frac{32}{2}\right)^2$

$x^2 + 32x + (16)^2$

① $\boxed{x^2 + 32x + 256}$

$(x+16)(x+16)$ or

② $\boxed{(x+16)^2}$ $\frac{b}{2}$

2) $c^2 - 16c + C$ $\boxed{b = -16}$

$c^2 - 16c + \left(\frac{b}{2}\right)^2$

$c^2 - 16c + \left(\frac{-16}{2}\right)^2$

$c^2 - 16c + (-8)^2$

① $\boxed{c^2 - 16c + 64}$

$(c-8)(c-8)$

or

② $\boxed{(c-8)^2}$ $\frac{b}{2}$

3) $8x + x^2$ $\boxed{b = 8}$

must be in DPO

$x^2 + 8x + C$

$x^2 + 8x + \left(\frac{b}{2}\right)^2$

$x^2 + 8x + \left(\frac{8}{2}\right)^2$

$x^2 + 8x + (4)^2$

① $\boxed{x^2 + 8x + 16}$

$(x+4)(x+4)$

or

② $\boxed{(x+4)^2}$ $\frac{b}{2}$

4) $w^2 - 13w + C$ $\boxed{b = -13}$

$w^2 - 13w + \left(\frac{b}{2}\right)^2$

$w^2 - 13w + \left(\frac{-13}{2}\right)^2$

$\boxed{w^2 - 13w + \frac{169}{4}}$

$\left(w - \frac{13}{2}\right)\left(w - \frac{13}{2}\right)$

or

② $\boxed{\left(w - \frac{13}{2}\right)^2}$ $\frac{b}{2}$

* Skip division when b is odd
* square numbers
* leave as improper fraction

5) $x^2 - 9x + \left(\frac{b}{2}\right)^2$

$b = -9$

$x^2 - 9x + \left(\frac{-9}{2}\right)^2$

* skip division
* square values
* leave as improper fraction

① $\boxed{x^2 - 9x + \frac{81}{4}}$

$\left(x - \frac{9}{2}\right)\left(x - \frac{9}{2}\right)^2$

or

② $\boxed{\left(x - \frac{9}{2}\right)^2}$ $\frac{b}{2}$

6) $x^2 + 22x + \left(\frac{22}{2}\right)^2 \rightarrow \left(\frac{b}{2}\right)^2$

$b = 22$

$\boxed{x^2 + 22x + 121}$

$\boxed{(x + 11)^2}$

$\frac{b}{2}$

7) $m^2 + 10m + \left(\frac{10}{2}\right)^2 \rightarrow \left(\frac{b}{2}\right)^2$

$b = 10$

$\boxed{m^2 + 10m + 25}$

$\boxed{(m + 5)^2}$

$\frac{b}{2}$

8) $x^2 - 5x + \left(\frac{-5}{2}\right)^2 \rightarrow \left(\frac{b}{2}\right)^2$

$b = -5$

$\boxed{x^2 - 5x + \frac{25}{4}}$

$\boxed{\left(x - \frac{5}{2}\right)^2}$

$\frac{b}{2}$

* skip division
* square values
* leave as improper fraction

9) $x^2 + 15x + \left(\frac{15}{2}\right)^2 \rightarrow \left(\frac{b}{2}\right)^2$

$b = 15$

$\boxed{x^2 + 15x + \frac{225}{4}}$

* skip division
* square values
* leave as improper fraction

$\boxed{\left(x + \frac{15}{2}\right)^2}$

$\frac{b}{2}$

10) $y^2 + 2y + \left(\frac{2}{2}\right)^2 \rightarrow \left(\frac{b}{2}\right)^2$

$b = 2$

$\boxed{y^2 + 2y + 1}$

$\boxed{(y + 1)^2}$

$\frac{b}{2}$

11) $b^2 + 4b + \left(\frac{4}{2}\right)^2 \rightarrow \left(\frac{b}{2}\right)^2$

$b = 4$

$\boxed{b^2 + 4b + 4}$

$\boxed{(b + 2)^2}$

$\frac{b}{2}$

** 12) $g^2 - 20g + 15$

$b = -20$

$g^2 - 20g + \left(\frac{-20}{2}\right)^2 + 15 - \left(\frac{-20}{2}\right)^2$

$g^2 - 20g + 100 + 15 - 100$

$\boxed{(g - 10)^2 - 85}$

$\frac{b}{2}$

13) $x^2 + 17x + 25$ $\left(\frac{b}{2}\right)^2$

$b = 17$

$x^2 + 17x + \left(\frac{17}{2}\right)^2 + 25 - \left(\frac{17}{2}\right)^2$

$x^2 + 17x + \frac{289}{4} + 25 - \frac{289}{4}$

* skip division
* square variables
* leave as improper fraction

$\boxed{\left(x + \frac{17}{2}\right)^2 - \frac{189}{4}}$

$\frac{b}{2}$

leave as improper fraction

14) $x^2 - 8x - 18$ $\left(\frac{b}{2}\right)^2$

$b = -8$

$x^2 - 8x + \left(\frac{-8}{2}\right)^2 - 18 - \left(\frac{-8}{2}\right)^2$

$x^2 - 8x + 16 - 18 - 16$

$\boxed{(x - 4)^2 - 34}$

$\frac{b}{2}$