

Do Now (# 1 & 2 only)

Solve for x:

1) $x^2 = 64$ ^{2 answers}
 $x = \pm 8$

2) $x^2 = 28$ ^{2 answers}
 $x = \pm 2\sqrt{7}$

$$x^2 = 49$$

$$\sqrt{x^2} = \pm\sqrt{49}$$

$$x = \pm 7$$

Perfect square on the left side (and right side in this problem).

When solving an equation involving a **perfect square** (such as $x^2 = 49$ seen at the left), taking the **square root of both sides** of the equation quickly yields the result.

This same square rooting process works nicely on equations with the square of a **binomial on the left side** (such as $(x+1)^2 = 9$ as seen at the right). We could even replace $(x+1)^2$ with $x^2 + 2x + 1$, and start with a **perfect square trinomial** on the left side.

$$(x+1)^2 = 9$$

$$\sqrt{(x+1)^2} = \pm\sqrt{9}$$

$$x+1 = \pm 3$$

$$x = -1 \pm 3$$

$$x = 2; x = -4$$

Statement:

Creating a **perfect square trinomial** on the left side of a quadratic equation, with a **constant (number)** on the right, is the basis of a method called **completing the square**.

In plain English, we are going to "force" a **perfect square trinomial** on the left hand side of the equation to help us find the solution more quickly.

$$|x+2| = 5$$

$$x+2 = 5 \quad x+2 = -5$$

Practice:

Solve for x:

3) $(x+2)^2 = 25$ ^{2 answers}
 $x+2 = \pm 5$
 $x+2 = 5$
 $\frac{-2 \quad -2}{x = 3}$
 $x+2 = -5$
 $\frac{-2 \quad -2}{x = -7}$
 $\{-7, 3\}$

4) $(x+4)^2 = 81$
 $x+4 = \pm 9$
 $x+4 = 9$
 $\frac{-4 \quad -4}{x = 5}$
 $x+4 = -9$
 $\frac{-4 \quad -4}{x = -13}$
 $\{-13, 5\}$

Solving Quadratics by Completing the Square
(where $a = 1$)

*You have solved quadratic equations by factoring and using the zero-product property. You can also solve quadratic equations by completing the square. This method is especially useful if the quadratic equation is difficult or impossible to factor.

*To solve a quadratic equation by completing the square, follow these steps:

<p>1. Check to see if the leading coefficient is one. If not, divide each term by the leading coefficient.</p>	$x^2 + 4x - 2 = 0$ <p>Leading coefficient is one.</p>
<p>2. If there is a constant term on the left side of the equation, move the constant term to the right side.</p>	$x^2 + 4x = 2$
<p>3. Set up the problem to receive the addition of the value which will create a perfect square trinomial on the left side. Inserting boxes may remind you to add the value to BOTH sides of the equation.</p>	$x^2 + 4x + \boxed{} = 2 + \boxed{}$ <p style="text-align: center;">$(\frac{b}{2})^2$ $(\frac{b}{2})^2$</p>
<p>4. To get the needed value for creating a perfect square trinomial, take half of the coefficient of the middle term (x-term) and square it. Add this value to both sides of the equation (put this value in the boxes).</p> <p style="text-align: center;">Take half and square</p> $x^2 + \textcircled{4}x + \boxed{} = 2 + \boxed{}$ <p style="text-align: center;">coefficient of "middle term"</p>	$x^2 + 4x + \boxed{4} = 2 + \boxed{4}$ <p>(Be sure to take note of the "sign" of half the coefficient of the middle term, as it will be used when factoring the perfect square trinomial. In this case, +2.)</p>
<p>5. Factor the perfect square trinomial on the left side.</p>	$(x + 2)^2 = 6$
<p>6. Now that there is a perfect square on the left side, take the square root of both sides. Solve for x. Be sure to remember to use "plus and minus" to arrive at the two roots of the equation.</p> <p>Check your solutions in the original equation to see that they work.</p> $\left. \begin{aligned} (-2 + \sqrt{6})^2 + 4(-2 + \sqrt{6}) - 2 &= 0 \\ 10 - 2\sqrt{6} - 8 + 2\sqrt{6} - 2 &= 0 \\ 0 &= 0 \text{ check} \end{aligned} \right\} \text{repeat using } -2 - \sqrt{6}$ <p>Always assume that answers are to be left in "exact" form (not rounded), unless told otherwise..</p>	$\sqrt{(x + 2)^2} = \pm\sqrt{6}$ $(x + 2) = \pm\sqrt{6}$ $x = -2 + \sqrt{6}$ $x = -2 - \sqrt{6}$ <p>In real-world problems and for graphing, these values may be expressed as rounded decimal values:</p> $x \approx 0.4494897428$ $x \approx -4.449489743$

AAA

Add to Both Side (Addition Property of Equality)

$\frac{b}{2}$

★ Constant Must be on the right side of = sign ★

Examples: Solve by completing the square. Check your answer.

$ax^2 + bx = c$

2 answers

1) $x^2 + 10x = -9$

$b=10$

$x^2 + 10x + (\frac{10}{2})^2 = -9 + (\frac{10}{2})^2$

$x^2 + 10x + 25 = -9 + 25$

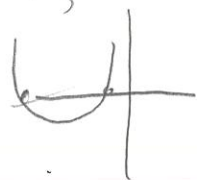
$\sqrt{(x+5)^2} = \sqrt{16}$

$x+5 = \pm 4$

$x+5=4 \quad | \quad x+5=-4$
 $-5 \quad -5 \quad | \quad -5 \quad -5$

$x = -1 \quad | \quad x = -9$

$\{-1, -9\}$



2) $x^2 + 25 = 10x$

$x^2 - 10x = -25$
 $-10x \quad -10x$

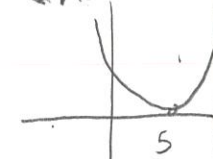
$x^2 - 10x + (\frac{10}{2})^2 = -25 + (\frac{10}{2})^2$

$x^2 - 10x + 25 = -25 + 25$

$\sqrt{(x-5)^2} = \sqrt{0}$

$x-5 = 0$ → only 1 equation b/c 0 is neutral
 $+5 \quad +5$
 $x = 5$

$\{5\}$



If bis odd
 * skip dividing
 * square #5
 leave as fraction

3) $t^2 - 8t - 5 = 0$

$t^2 - 8t = 5$
 $+16 \quad +16$
 $t^2 - 8t + 16 = 5 + 16$

$\sqrt{(t-4)^2} = \sqrt{21}$

$t-4 = \pm \sqrt{21}$

$t-4 = \sqrt{21} \quad | \quad t-4 = -\sqrt{21}$
 $+4 \quad +4 \quad | \quad +4 \quad +4$

$t = 4 + \sqrt{21} \quad | \quad t = 4 - \sqrt{21}$

$\{4 + \sqrt{21}, 4 - \sqrt{21}\}$

$t = 4 \pm \sqrt{21}$

4) $x^2 - 2x - 7 = 0$

$x^2 - 2x = 7$
 $+1 \quad +1$
 $x^2 - 2x + (\frac{-2}{2})^2 = 7 + (\frac{-2}{2})^2$

$x^2 - 2x + 1 = 7 + 1$
 $\sqrt{(x-1)^2} = \sqrt{8}$

$x-1 = \pm \sqrt{8}$
 $\sqrt{4} \cdot \sqrt{2}$

$x-1 = \pm 2\sqrt{2}$
 $x-1 = 2\sqrt{2} \quad | \quad x-1 = -2\sqrt{2}$
 $+1 \quad +1 \quad | \quad +1 \quad +1$

$x = 1 + 2\sqrt{2} \quad | \quad x = 1 - 2\sqrt{2}$

$\{1 + 2\sqrt{2}, 1 - 2\sqrt{2}\}$

$x = 1 \pm 2\sqrt{2}$

5) $p^2 - 3p = 18$

$p^2 - 3p + (\frac{-3}{2})^2 = 18 + (\frac{-3}{2})^2$

$p^2 - 3p + \frac{9}{4} = 18 + \frac{9}{4}$
 $\sqrt{(p - \frac{3}{2})^2} = \sqrt{\frac{81}{4}}$ → leave as improper

$p - \frac{3}{2} = \pm \sqrt{\frac{81}{4}}$
 $p - \frac{3}{2} = \pm \frac{\sqrt{81}}{\sqrt{4}}$

$p - \frac{3}{2} = \pm \frac{9}{2}$
 $p - \frac{3}{2} = \frac{9}{2} \quad | \quad p - \frac{3}{2} = -\frac{9}{2}$
 $+ \frac{3}{2} \quad + \frac{3}{2} \quad | \quad + \frac{3}{2} \quad + \frac{3}{2}$

$p = 6 \quad | \quad p = -3$

$\{6, -3\}$

6) $x^2 - 6x = -10$

$x^2 - 6x + (\frac{-6}{2})^2 = -10 + (\frac{-6}{2})^2$

$x^2 - 6x + 9 = -10 + 9$
 $\sqrt{(x-3)^2} = \sqrt{-1}$ → can't have a negative radicand

$x-3 = \pm \sqrt{-1}$
 $x-3 = \sqrt{-1} \quad | \quad x-3 = -\sqrt{-1}$
 $+3 \quad +3 \quad | \quad +3 \quad +3$

$x = 3 + \sqrt{-1} \quad | \quad x = 3 - \sqrt{-1}$

$\{3 + \sqrt{-1}, 3 - \sqrt{-1}\}$

$x = 3 \pm \sqrt{-1}$
 NO Real roots b/c it's imaginary

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