

Relations and Functions

8.A.17 Define and use correct terminology when referring to function (domain and range)

8.A.18 Determine if a relation is a function

DRY MIX

(3, 4)

★ Key ★

An **ordered pair** names the location of a point on a coordinate grid. The x -coordinate (the **first coordinate**) tells you how far to the right or left of 0 the point is, and the y -coordinate tells you how far up or down from 0 the point is.

A **relation** is a set of ordered pairs, for example, $\{(-4, 3), (4, 9), (5, 12)\}$.

The **domain** of a relation is the set of x -values of the relation. The **range** of a relation is the set of y -values of the relation. For the relation shown above, the domain is $\{-4, 4, 5\}$ and the range is $\{3, 9, 12\}$.

*Dx
Ry*

A **function** is a relation in which each element of the domain corresponds to one and only one element of the range. In terms of ordered pairs, a function is a relation in which **no two ordered pairs have the same x -coordinate**. The relation above has no two x -coordinates that are the same, so that relation is a function.

*★ X-values don't repeat
★ y-values may repeat*

Is the following relation a function? Why or why not?

$\{(1, 5), (2, 10), (3, 15), (4, 20)\}$

Use the definition of function.

STEP 1 List the x -coordinates.

1, 2, 3, 4

STEP 2 Do any of the x -coordinates repeat?

Each x -coordinate appears only once. Another way of stating this is that each x -coordinate (domain value) is paired with exactly one y -coordinate (range value).

The relation **is a function** because each member of the domain is paired with exactly one member of the range.

y

X-values

① Domain

② Input

③ Independent

Y-values

① Range

② Output

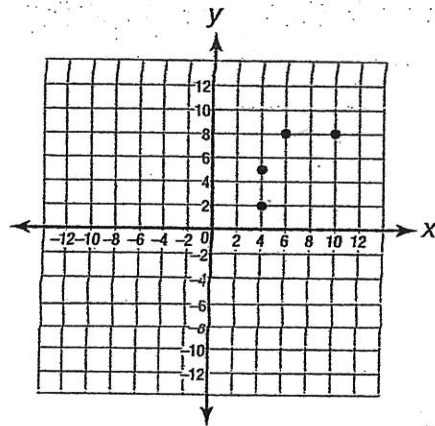
③ Dependent

EXAMPLE 1

STRATEGY

SOLUTION

Relations and functions are sets of ordered pairs, so they can be graphed on a coordinate plane. Here is a graph of the relation $\{(4,2), (4,5), (6,8), (10,8)\}$.

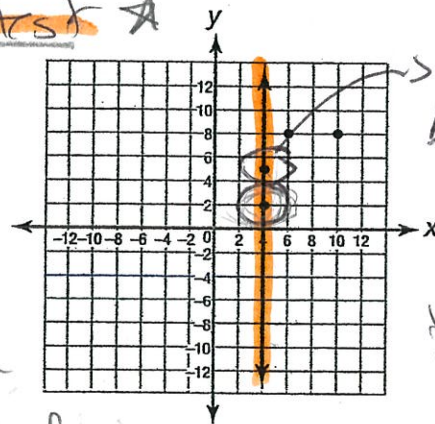


\rightarrow x's repeat

If the graph of a relation has more than one y-coordinate for an x-coordinate, then the relation is not a function. There is an easy way to check if there is more than one y-coordinate for each x-coordinate. If you can draw a vertical line through more than one point on the graph, then the relation is not a function.

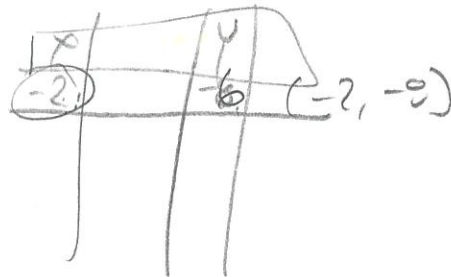
A vertical line test

- * you use when you are given a graph
- * once = is a function
- * twice or more = NOT a function



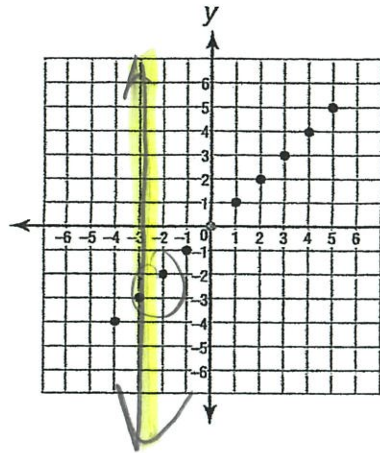
NOT a function
B/c the vertical line intersects the graph twice
(Fails vertical line test)

A vertical line can be drawn through two points, so the relation $\{(4,2), (4,5), (6,8), (10,8)\}$ is not a function.



EXAMPLE 2

Does the graph show a function? If so, why?



*PASSES
the
vertical
line test
It is a function*

STRATEGY

Use the vertical line test.

It is not possible to draw a vertical line through more than one point.

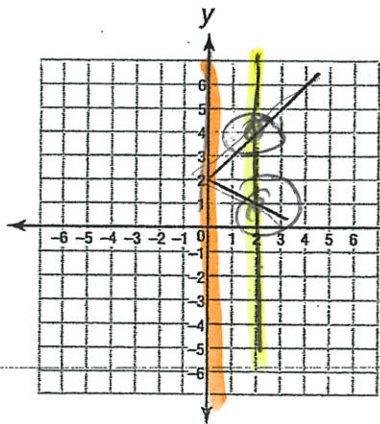
SOLUTION

The graph shows a function because it is impossible to draw a vertical line through more than one point.

The relation in Example 2 has a finite (countable) number of points. The vertical line test can also be used for relations that consist of an infinite number of points.

EXAMPLE 3

The following graph shows a relation with an infinite number of points. Does the graph represent a function? Why or why not?

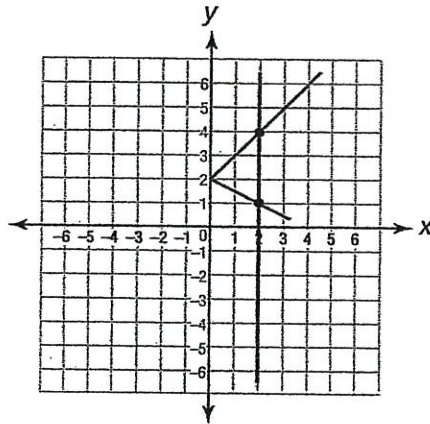


*NOT a
function
because the
x value
repeats
fails the
vertical line test*

STRATEGY

Use the vertical line test.

You can draw a vertical line through two points at many locations. One example would be the points at (2,1) and (2,4).



SOLUTION

The graph is not a function because it is possible to draw a vertical line that intersects the graph in more than one point.



Is this relation a function?

$\{(-2,4), (-1,1), (0,0), (1,1), (2,4)\}$

Let's check it out.

The domain of the relation is $\{-2, -1, 0, 1, 2\}$

The range of the relation is $\{0, 1, 4\}$

Do any of the elements of the domain repeat? NO

Explain why the relation is or is not a function. Yes!

(B/c the x-values don't repeat)

- ~~Demand list~~
- * Numerical order
- * Don't list the repeats
- * Use braces { }

they can repeat

yes

can repeat

Always look at the original coordinates to see if something repeats

Sample Test Questions

1 What is the range of this relation?

$\{(1,2), (3,4), (5,6), (7,8), (7,10)\}$

- A {1, 3, 5, 7, 8}
- B {5, 6, 7, 8, 10}
- C {2, 4, 6, 7, 8}
- D {2, 4, 6, 8, 10}

2 Which relation is a function?

- ~~F~~ $\{(1,3), (2,2), (3,3), (3,4)\}$
- ~~G~~ $\{(10,15), (20,40), (20,45), (30,50)\}$
- H $\{(12,28), (14,28), (16,30), (18,32)\}$
- J $\{(22,11), (24,12), (26,12), (26,13)\}$

*x-values
Don't
repeat*

3 Which relation is a function?

- A $\{(-1,1), (2,8), (-1,-1), (-2,-8)\}$
- B $\{(1,2), (2,8), (3,18), (2,32)\}$
- C $\{(-3,9), (-2,4), (2,4), (3,9)\}$
- D $\{(1,5), (2,10), (-2,10), (1,15)\}$

*x-values
Don't repeat*

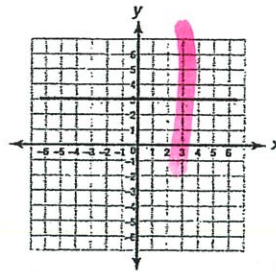
4 Which relation is not a function?

- F $\{(\frac{1}{2}, \frac{1}{3}), (\frac{1}{4}, \frac{1}{5}), (\frac{1}{6}, \frac{1}{7}), (\frac{1}{8}, \frac{1}{9})\}$
- G $\{(\frac{1}{2}, \frac{2}{2}), (\frac{1}{3}, \frac{3}{2}), (\frac{2}{3}, \frac{2}{3}), (\frac{3}{3}, \frac{4}{3})\}$
- H $\{(\frac{1}{2}, \frac{1}{5}), (\frac{3}{2}, \frac{1}{6}), (\frac{4}{2}, \frac{1}{7}), (\frac{5}{2}, \frac{1}{8})\}$
- J $\{(\frac{1}{2}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{8}), (\frac{4}{8}, \frac{1}{16}), (\frac{4}{16}, \frac{1}{32})\}$

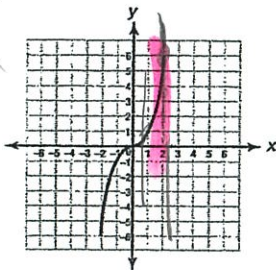
*x-values
DO
repeat*

5 Which graph below does not show a function?

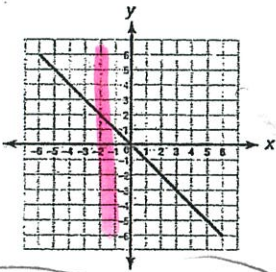
A



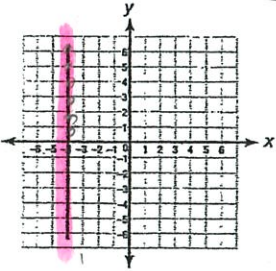
B



C

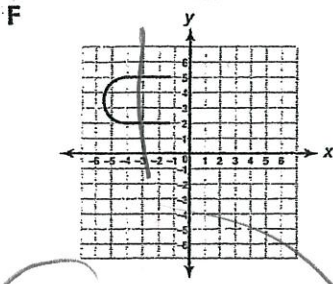


D

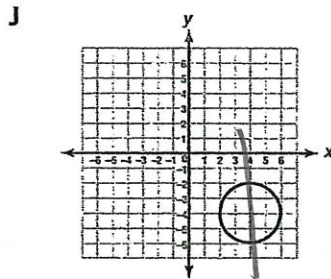
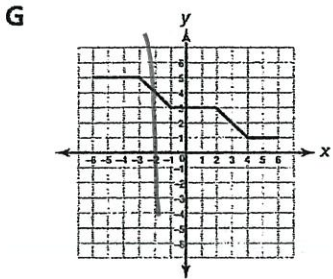
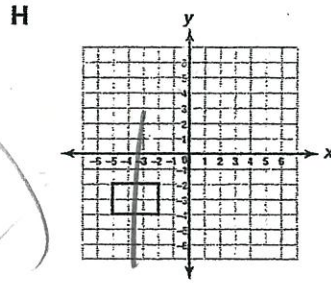


*Fails
vertical
line test
(goes through
more than
one point)*

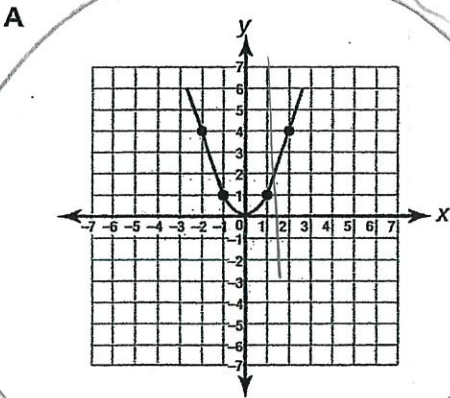
6 Which graph shows a function?



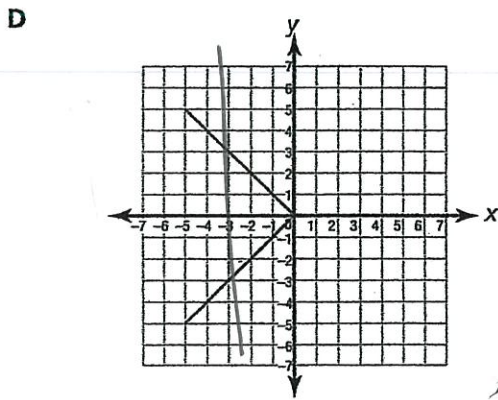
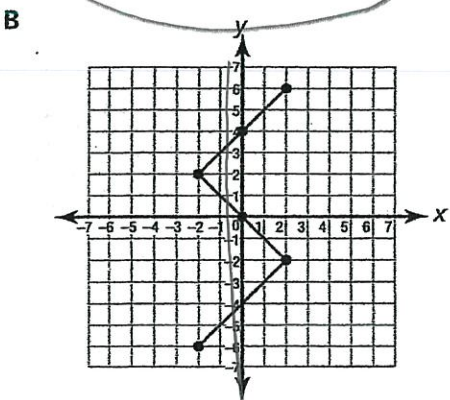
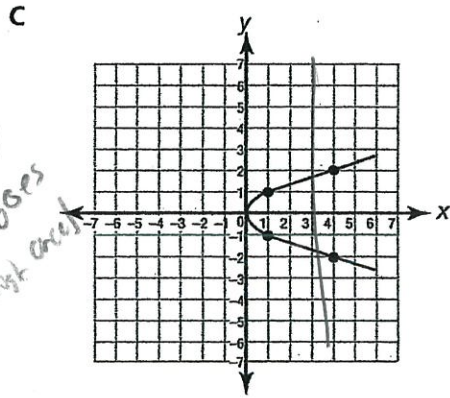
passes vertical line test (only goes through once)



7 Which graph shows a function?



passes the vertical line test (only goes through once)



8 Write the **domain** of this relation.

$\{(5,25), (6,36), (-5,25), (-6,36)\}$

Answer: $\{-6, -5, 5, 6\}$
 Use Braces $\{ \}$
 Numerical order

No repeats!

9 Write the **range** of this relation.

$\{(2,20), (2,-40), (-2,10), (-2,-30)\}$

Answer: $\{-40, -30, 10, 20\}$
 Use Braces $\{ \}$
 Numerical order

No repeats!

Extended-Response Question

10

Part A

Determine if the relation below is a function. Explain your answer.

$\{(-3,8), (-2,3), (-1,0), (0,-1), (1,0), (2,3), (3,8)\}$

Answer: yes! b/c the x-values do not repeat

Part B

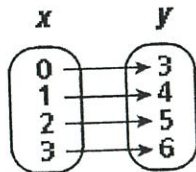
How could you explain your answer to Part A using a graph?

A vertical line only goes through the graph once so it passes the vertical line test

mapping

Function ✓

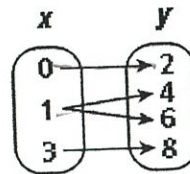
Each input gives only one output.



$(0,3)$
 $(1,4)$
 $(2,5)$
 $(3,6)$

Not a Function

One input gives more than one output.



$(0,2)$
 $(1,4)$
 $(1,6)$
 $(3,8)$ } is repeat

Max! ☺

X-values DO NOT repeat or pass the vertical line test

Are the following functions?

a)

X	Y
-1	-7
9	1
12	8
15	-7

Yes!
X-values don't repeat

b)

X	Y
2	4
5	5
8	6
2	7

NO!
the 2's repeat (X-values repeat)

c) $\{(1,3), (2,6), (1,3)\}$
 $\{2,6\}$
 exact same pm
 Yes!
 NO X-values repeat

