

Discriminant

The **discriminant** is the name given to the expression that appears under the square root (radical) sign in the quadratic formula.



<p>Quadratic Formula:</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	<p>Discriminant</p> $b^2 - 4ac$ <p>* NO $\sqrt{\quad}$ sign</p>
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The discriminant tells you about the "nature" of the roots of a quadratic equation given that a , b and c are rational numbers. It quickly tells you the number of real roots, or in other words, the number of x -intercepts, associated with a quadratic equation.

* Equation \rightarrow must be in standard form ($= 0$ & DPO)

② Solutions
③ Zeros

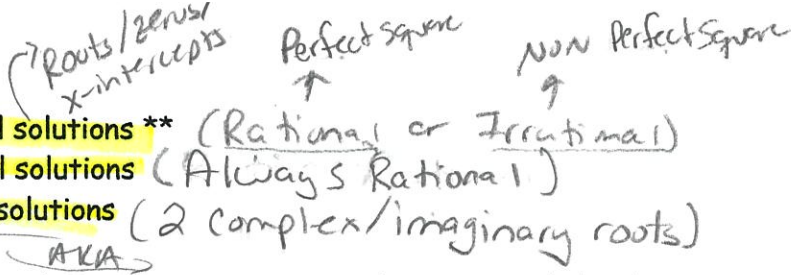
Equation	$x^2 + 6x + 5 = 0$ $a = 1, b = 6, c = 5$	$x^2 - 2x + 1 = 0$ $a = 1, b = -2, c = 1$	$x^2 - 3x + 10 = 0$ $a = 1, b = -3, c = 10$
Discriminant	$b^2 - 4ac$ $(6)^2 - 4(1)(5)$ $36 - 20$ 16	$b^2 - 4ac$ $(-2)^2 - 4(1)(1)$ $4 - 4$ 0	$b^2 - 4ac$ $(-3)^2 - 4(1)(10)$ $9 - 40$ -31
Number of Solutions	Two real solutions	One real solution	No real solutions imaginary/NOT Real
Graph of Roots	<p>intersects the x-axis twice</p> <p>$x = -5$ $x = -1$</p>	<p>graph only intersects the x-axis one time</p> <p>$x = 1$</p>	<p>DOES NOT intersect the x-axis at all</p> <p>NO REAL ROOTS</p>
Roots found using the Quadratic Equation	$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$ $x = \frac{-6 \pm \sqrt{16}}{2} = \frac{-6 \pm 4}{2}$ $x = -1; x = -5$	$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$ $x = \frac{2 \pm \sqrt{0}}{2} = 1$ $x = 1; x = 1$	$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(10)}}{2(1)}$ $x = \frac{3 \pm \sqrt{-31}}{2}$ $x = \frac{3 + i\sqrt{31}}{2}; x = \frac{3 - i\sqrt{31}}{2}$
	<p>① Rational = perfect squares</p> <p>② Irrational = non-perfect squares</p> <p>There are two real roots</p> <p>① Rational: # under $\sqrt{\quad}$ (or discriminant) is a perfect square</p> <p>② Irrational: # under $\sqrt{\quad}$ (or discriminant) is not a perfect square</p>	<p>There is one real root</p> <p>Always Rational</p>	<p>There are two complex roots.</p> <p>imaginary/NOT Real</p>

So....

If $b^2 - 4ac > 0$, the equation has two real solutions ** (Rational or Irrational)

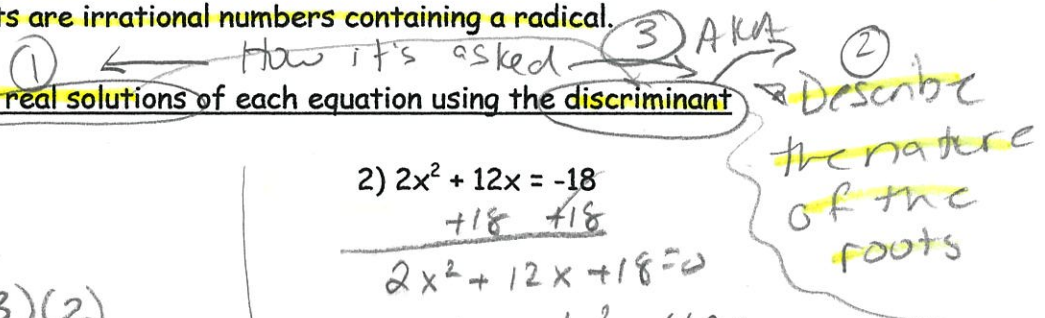
If $b^2 - 4ac = 0$, the equation has one real solutions (Always Rational)

If $b^2 - 4ac < 0$, the equation has no real solutions (2 complex/imaginary roots)



**If the discriminant is a perfect square, the two roots are rational numbers. If the discriminant is not a perfect square, the two roots are irrational numbers containing a radical.

Examples: Find the number of real solutions of each equation using the discriminant



1) $3x^2 + 10x + 2 = 0$

$a = 3$
 $b = 10$
 $c = 2$

$$b^2 - 4ac$$

$$(10)^2 - 4(3)(2)$$

$$100 - 4(3)(2)$$

$$100 - 24$$

$$\boxed{76} \rightarrow \text{NON P.S.}$$

2 real irrational roots/solutions

2) $2x^2 + 12x = -18$

$$\frac{2x^2 + 12x = -18}{+18 \quad +18}$$

$$2x^2 + 12x + 18 = 0$$

$a = 2$
 $b = 12$
 $c = 18$

$$b^2 - 4ac$$

$$(12)^2 - 4(2)(18)$$

$$144 - 4(2)(18)$$

$$144 - 144$$

$$\boxed{0} \rightarrow \text{P.S.}$$

1 real rational root/solution

3) $x^2 + x + 1 = 0$

$a = 1$
 $b = 1$
 $c = 1$

$$b^2 - 4ac$$

$$(1)^2 - 4(1)(1)$$

$$1 - 4(1)(1)$$

$$1 - 4$$

$$\boxed{-3}$$

2 complex/imaginary roots/solutions, NO Real Roots

4) $x^2 + 4x - 5 = 0$

$a = 1$
 $b = 4$
 $c = -5$

$$b^2 - 4ac$$

$$(4)^2 - 4(1)(-5)$$

$$16 - 4(1)(-5)$$

$$16 + 20$$

$$\boxed{36} \rightarrow \text{perfect square!}$$

2 real rational roots/solutions

5) $2x^2 - 2x + 3 = 0$

$a = 2$
 $b = -2$
 $c = 3$

$$b^2 - 4ac$$

$$(-2)^2 - 4(2)(3)$$

$$4 - 4(2)(3)$$

$$4 - 24$$

$$\boxed{-20}$$

2 complex/imaginary roots/solutions, NO Real Roots

6) $9x^2 - 7x + 1 = 0$

$a = 9$
 $b = -7$
 $c = 1$

$$b^2 - 4ac$$

$$(-7)^2 - 4(9)(1)$$

$$49 - 4(9)(1)$$

$$49 - 36$$

$$\boxed{13} \rightarrow \text{NON-perfect square}$$

2 real irrational roots/solutions