



How Do We Solve A System Of Linear Equations Algebraically?

Homework #36-8

★ Part I: Addition Method ★

★ aka: Elimination Method ★

Procedure:

1. Make sure the two equations are in proper form by making sure that all variables are on one side of the equal sign and all constants are on the other side.
2. Make sure to line up the two equations appropriately. (Variable on top of corresponding variable, equal sign on top of equal sign, constant on top of constant.)  $6 \rightarrow -6$   
 $-7 \rightarrow 7$
3. Determine if the system contains additive inverses. Use multipliers if needed.
4. Combine the two equations by adding, thus reducing the problem to one equation in one variable.
5. Solve for the one variable.
6. Substitute the known value in any of the two original equations and solve for the second variable.
7. Check both values in each of the original equations.

Examples: Solve the following systems of equations using the addition method. Check your answer.

$$\begin{array}{l} 1) \quad \left\{ \begin{array}{l} 3x - y = 7 \\ 2x + y = 8 \end{array} \right. \rightarrow \begin{array}{l} 6x - 2y = 14 \\ + \quad 2x + y = 8 \\ \hline 8x - y = 22 \end{array} \\ \rightarrow \begin{array}{l} 6x - 2y = 14 \\ - \quad 6x - 3y = -24 \\ \hline -5y = -10 \\ -5 \quad -5 \\ \hline y = 2 \end{array} \end{array}$$

$(3, 2)$   
x y

$$\begin{array}{l} 2x + y = 8 \\ 2x + (2) = 8 \\ \hline -2 - 2 \\ \hline 2x = 6 \\ \frac{2}{2} \quad \frac{2}{2} \\ \hline x = 3 \end{array}$$

Check #1

$$\begin{array}{l} (3, 2) \\ \begin{array}{l} 3x - y = 7 \\ 3(3) - (2) = 7 \\ 9 - 2 = 7 \\ 7 = 7 \end{array} \end{array}$$

Check #2

$$\begin{array}{l} (3, 2) \\ \begin{array}{l} 2x + y = 8 \\ 2(3) + 2 = 8 \\ 6 + 2 = 8 \\ 8 = 8 \end{array} \end{array}$$

$$\begin{array}{l} 2) \quad \left\{ \begin{array}{l} 5x - 2y = 20 \\ 2x + 3y = 27 \end{array} \right. \rightarrow \begin{array}{l} 10x - 4y = 40 \\ + \quad 10x - 15y = -135 \\ \hline -19y = -95 \\ -19 \quad -19 \\ \hline y = 5 \end{array} \end{array}$$

$(6, 5)$

$y = 5$

$$\begin{array}{l} 2x + 3y = 27 \\ 2x + 3(5) = 27 \\ 2x + 15 = 27 \\ \hline -15 - 15 \\ \hline 2x = 12 \\ \frac{2}{2} \quad \frac{2}{2} \\ \hline x = 6 \end{array}$$

Check #1

$$\begin{array}{l} (6, 5) \\ \begin{array}{l} 5x - 2y = 20 \\ 5(6) - 2(5) = 20 \\ 30 - 10 = 20 \\ 20 = 20 \end{array} \end{array}$$

Check #2

$$\begin{array}{l} (6, 5) \\ \begin{array}{l} 2x + 3y = 27 \\ 2(6) + 3(5) = 27 \\ 12 + 15 = 27 \\ 27 = 27 \end{array} \end{array}$$

$$3) \begin{cases} 4(5a + b = 13) \rightarrow 20a + 4b = 52 \\ -5(4a - 3b = 18) \rightarrow -20a + 15b = -90 \end{cases}$$

$$\begin{array}{r} 19b = -38 \\ \hline 19 \quad 19 \\ \hline b = -2 \end{array}$$

$$(3, -2)$$

$$\begin{array}{l} 5a + b = 13 \\ 5a + (-2) = 13 \\ 5a - 2 = 13 \\ \hline 5a = 15 \\ \hline 5 \quad 5 \\ \hline a = 3 \end{array}$$

Check #1

$$\begin{array}{l} (3, -2) \\ \begin{matrix} a \\ b \end{matrix} \\ 5a + b = 13 \\ 5(3) + (-2) = 13 \\ 15 - 2 = 13 \\ 13 = 13 \end{array}$$

Check #2

$$\begin{array}{l} (3, -2) \\ \begin{matrix} a \\ b \end{matrix} \\ 4a - 3b = 18 \\ 4(3) - 3(-2) = 18 \\ 12 + 6 = 18 \\ 18 = 18 \end{array}$$

$$4) \begin{cases} 3(2x + 5y = -1) \rightarrow 6x + 15y = -3 \\ -2(-3x + y = 10) \rightarrow 6x + 2y = 20 \end{cases}$$

Don't add negative if it is already there and don't bring up or down a negative either

$$\begin{array}{r} 17y = 17 \\ \hline 17 \quad 17 \\ \hline y = 1 \end{array}$$

$$(-3, 1)$$

$$\begin{array}{l} 2x + 5y = -1 \\ 2x + 5(1) = -1 \\ 2x + 5 = -1 \\ \hline 2x = -6 \\ \hline 2 \quad 2 \\ \hline x = -3 \end{array}$$

Check #1

$$\begin{array}{l} (-3, 1) \\ 2x + 5y = -1 \\ 2(-3) + 5(1) = -1 \\ -6 + 5 = -1 \\ -1 = -1 \end{array}$$

Check #2

$$\begin{array}{l} (-3, 1) \\ -3x + y = 10 \\ -3(-3) + (1) = 10 \\ 9 + 1 = 10 \\ 10 = 10 \end{array}$$

$$5) \begin{cases} 3a - 7 = 7b \\ 4a = 3b + 22 \end{cases}$$

$$\begin{array}{r} 3a - 7 = 7b \\ -7b \quad 7b \\ \hline 3a - 7b - 7 = 0 \\ \hline 3a - 7b = 7 \end{array}$$

$$\begin{array}{r} 4a = 3b + 22 \\ -3b \quad 3b \\ \hline 4a - 3b = 22 \end{array}$$

need to be in this for

$$\begin{cases} 4(3a - 7b = 7) \rightarrow 12a - 28b = 28 \\ -3(4a - 3b = 22) \rightarrow -12a + 9b = -66 \end{cases}$$

$$\begin{array}{r} -19b = -38 \\ \hline -19 \quad 19 \\ \hline b = 2 \end{array}$$

$$\begin{array}{l} 4a = 3b + 22 \\ 4a = 3(2) + 22 \\ 4a = 6 + 22 \\ 4a = 28 \\ \hline 4 \quad 4 \\ \hline a = 7 \end{array}$$

$$(7, 2)$$

Check #1

$$\begin{array}{l} (7, 2) \\ \begin{matrix} a \\ b \end{matrix} \\ 3a - 7 = 7b \\ 3(7) - 7 = 7(2) \\ 21 - 7 = 14 \\ 14 = 14 \end{array}$$

Check #2

$$\begin{array}{l} (7, 2) \\ \begin{matrix} a \\ b \end{matrix} \\ 4a = 3b + 22 \\ 4(7) = 3(2) + 22 \\ 28 = 6 + 22 \\ 28 = 28 \end{array}$$

6)  $7x = 5 - 2y$   
 $3y = 16 - 2x$

$$\begin{array}{r} 7x = 5 - 2y \\ +2y \quad +2y \\ \hline 7x + 2y = 5 \end{array}$$

$$\begin{array}{r} 3y = 16 - 2x \\ +2x \quad +2x \\ \hline 2x + 3y = 16 \end{array}$$

2  $(7x + 2y = 5) \rightarrow 14x + 4y = 10$   
 $- (2x + 3y = 16) \rightarrow -14x - 21y = -112$

$$\begin{array}{r} 14x + 4y = 10 \\ -14x - 21y = -112 \\ \hline -17y = -102 \\ \frac{-17y}{-17} = \frac{-102}{-17} \\ y = 6 \end{array}$$

$$\begin{array}{r} 7x = 5 - 2y \\ 7x = 5 - 2(6) \\ 7x = 5 - 12 \\ 7x = -7 \\ \frac{7x}{7} = \frac{-7}{7} \\ x = -1 \end{array}$$

$(-1, 6)$

Check #1  
 $(-1, 6)$   
 $xy$   
 $7x = 5 - 2y$   
 $7(-1) = 5 - 2(6)$   
 $-7 = 5 - 12$   
 $-7 = -7$   
 ✓

Check #2  
 $(-1, 6)$   
 $xy$   
 $3y = 16 - 2x$   
 $3(6) = 16 - 2(-1)$   
 $18 = 16 + 2$   
 $18 = 18$   
 ✓

7)  $5m + 3n = 14$   
 $-5(2m + n = 6) \rightarrow -10m - 5n = -30$

$$\begin{array}{r} 5m + 3n = 14 \\ -10m - 5n = -30 \\ \hline n = -2 \end{array}$$

$(4, -2)$

$5m + 3n = 14$   
 $5m + 3(-2) = 14$   
 $5m - 6 = 14$   
 $+6 \quad +6$   
 $5m = 20$   
 $\frac{5m}{5} = \frac{20}{5}$   
 $m = 4$

check #1  
 $(4, -2)$   
 $5m + 3n = 14$   
 $5(4) + 3(-2) = 14$   
 $20 - 6 = 14$   
 $14 = 14$   
 ✓

check #2  
 $(4, -2)$   
 $2m + n = 6$   
 $2(4) + (-2) = 6$   
 $8 - 2 = 6$   
 $6 = 6$   
 ✓

8)  $4x + 3y = 12$   
 $4(-2x + y = -16) \rightarrow -8x + 4y = -64$

→ Don't multiply by the negative because it's already a (ready)

$$\begin{array}{r} 4x + 3y = 12 \\ -8x + 4y = -64 \\ \hline \frac{10y}{10} = \frac{-40}{10} \\ y = -4 \end{array}$$

$(6, -4)$

$4x + 3y = 12$   
 $4x + 3(-4) = 12$   
 $4x - 12 = 12$   
 $+12 \quad +12$   
 $4x = 24$   
 $\frac{4x}{4} = \frac{24}{4}$   
 $x = 6$

check #1  
 $(6, -4)$   
 $4x + 3y = 12$   
 $4(6) + 3(-4) = 12$   
 $24 - 12 = 12$   
 $12 = 12$   
 ✓

check #2  
 $(6, -4)$   
 $xy$   
 $-2x + y = -1$   
 $-2(6) + (-4) = -1$   
 $-12 - 4 = -1$   
 $-16 = -16$   
 ✓

14



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