

Name \_\_\_\_\_

Date \_\_\_\_\_

## How Do We Know If A Relation Is A Function? How Do We Identify The Domain And Range Of A Function?

### VOCABULARY

A **relation** is a set of ordered pairs.

The set of first coordinates from the ordered pairs is the **domain**.

The set of second coordinates from the ordered pairs is the **range**.

A **function** is a relation in which each domain value is paired with one range value.

The **vertical line test** states that the graph of a relation is a function if every vertical line passes through no more than one point on the graph.

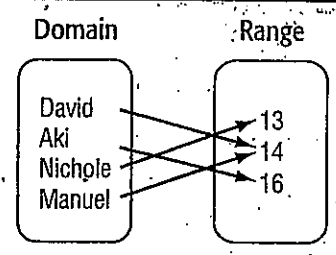
### KEY CONCEPT: Identifying the Domain and Range of a Relation

Each element of the domain is paired with an element of the range: (Domain, Range).

**{{(David, 14), (Aki, 16), (Nichole, 13), (Manuel, 14)}**

**Domain:** {David, Aki, Nichole, Manuel}

**Range:** {13, 14, 16}



An **ordered pair** names the location of a point on a coordinate grid.

The  $x$ -coordinate (the first coordinate) tells you how far to the right or left of 0 the point is, and the  $y$ -coordinate tells you how far up or down from 0 the point is.

A **relation** is a set of ordered pairs, for example,  $\{(-4,3), (4,9), (5,12)\}$ .

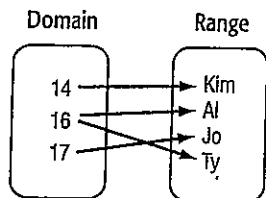
The **domain** of a relation is the set of  $x$ -values of the relation. The **range** of a relation is the set of  $y$ -values of the relation. For the relation shown above, the domain is  $\{-4, 4, 5\}$  and the range is  $\{3, 9, 12\}$ .

A **function** is a relation in which each element of the domain corresponds to one and only one element of the range. In terms of ordered pairs, a function is a relation in which no two ordered pairs have the same  $x$ -coordinate. The relation above has no two  $x$ -coordinates that are the same, so that relation is a function.

A relation in which the first coordinate is never repeated is called a *function*.

### Not a Function

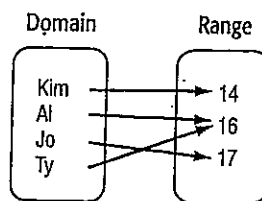
$\{(14, \text{Kim}), (16, \text{Al}), (17, \text{Jo}), (16, \text{Ty})\}$



In a function, each domain value (age) can have only one range value (students). The domain value 16 has two range values: Al and Ty.

### Function

$\{(\text{Kim}, 14), (\text{Al}, 16), (\text{Jo}, 17), (\text{Ty}, 16)\}$



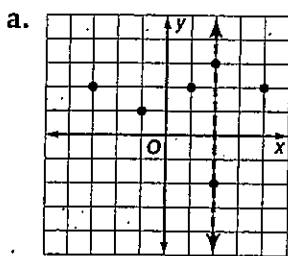
If we switch the domain and range, the new relation is a function. Each domain value (student) has *exactly* one range value (age).

You can identify whether a given relation is a function by inspecting its ordered pairs.

## Vertical Line Test

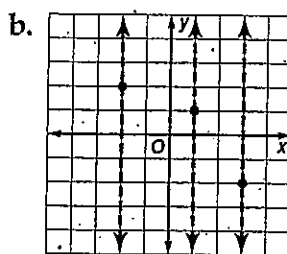
The graph of a relation on the coordinate plane can help you visually identify whether a relation is a function. In an ordered pair, the domain value is the  $x$ -coordinate and the range value is the  $y$ -coordinate. Because every point on a vertical line has the same  $x$ -coordinate, a vertical line can never be a function. The *vertical line test* states that a relation is a function if no vertical line passes through more than one point on the graph.

Use the vertical line test to identify whether each relation is a function.



This relation is *not* a function. At least one vertical line passes through two points.

**Understanding the Solution** Notice that  $(2, 3)$  and  $(2, -2)$  both have the same  $x$ -coordinate, 2.

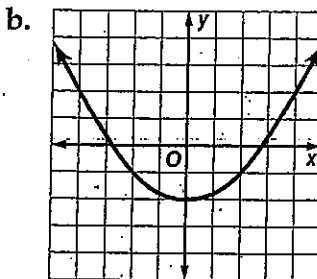
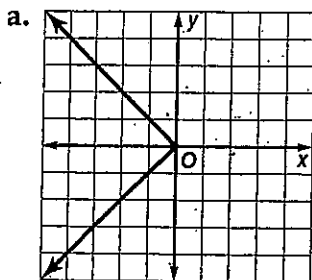


This relation is a function. No vertical line passes through more than one point on the graph.

**Understanding the Solution** No points have the same  $x$ -coordinate.

### TRY IT!

Use the vertical line test to identify whether each relation is a function.



## Functional Notation:

Traditionally, functions are referred to by the notation  $f(x)$ , but  $f$  need not be the only letter used in function names. Remember that  $f(x)$  is telling you that the result will be "a function of  $x$ ", or is dependent upon  $x$ . The statements  $y = x^2$  and  $f(x) = x^2$  are basically the same.

You may even see statements such as  $f(x) = y = x^2$ .

**Example:** A function is represented by  $f(x) = 2x + 5$ . Find  $f(3)$ .

To find  $f(3)$ , replace the  $x$ -value with 3.  $f(3) = 2(3) + 5 = 11$ .

The answer, 11, is called the image of 3 under  $f(x)$ .

**Note:** The  $f(x)$  notation can be thought of as another way of representing the  $y$ -value, especially when graphing. The  $y$ -axis may even be labeled as the  $f(x)$  axis.

## Exercises

### MULTIPLE CHOICE

1. What is the domain of the relation:  $\{(0, 1), (2, 1), (1, 3), (2, 4)\}$ ?

- A  $\{0, 1, 3\}$
- B  $\{1, 3, 4\}$
- C  $\{0, 1, 2\}$
- D  $\{0, 1, 2, 2\}$

2. What is the range of the function:  $\{(6, 4), (3, 8), (7, 9), (9, 8)\}$ ?

- A  $\{4, 8, 9\}$
- B  $\{3, 4, 6, 8\}$
- C  $\{6, 7, 8, 9\}$
- D  $\{3, 6, 7, 9\}$

3. Which of the following relations is *not* a function?

A	$x$	1	2	3	4
	$y$	2	4	6	8
B	$x$	0	2	4	6
	$y$	2	4	8	10
C	$x$	1	2	3	4
	$y$	2	2	2	2
D	$x$	1	2	4	4
	$y$	8	6	4	2

4. Which of the following relations is a function?

- A  $\{(4, 6), (7, 1), (9, 5), (7, 5)\}$
- B  $\{(4, 6), (5, 1), (9, 5), (7, 5)\}$
- C  $\{(4, 6), (5, 1), (5, 9), (7, 5)\}$
- D  $\{(4, 6), (5, 1), (4, 3), (7, 5)\}$

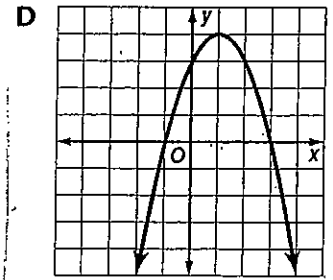
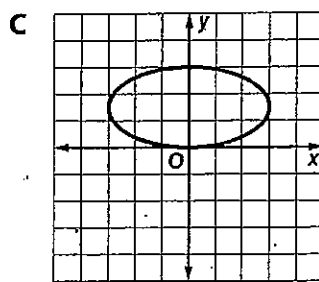
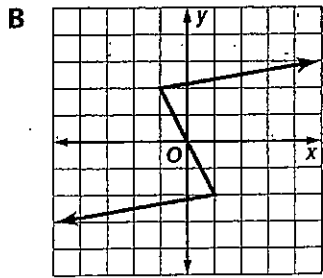
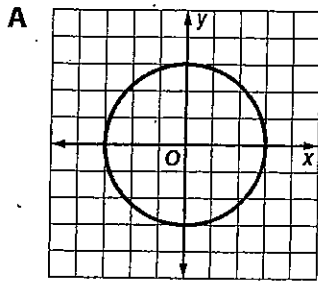
5. Which of the following relations is *not* a function?

- A  $\{(-4, -3), (2, 0), (4, 3), (5, -4)\}$
- B  $\{(4, -3), (2, 0), (-4, 3), (-5, 4)\}$
- C  $\{(-4, -3), (-2, 0), (2, 3), (4, 3)\}$
- D  $\{(-4, -3), (2, 0), (4, 3), (2, -4)\}$

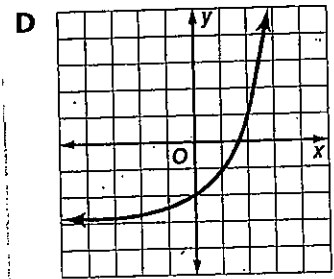
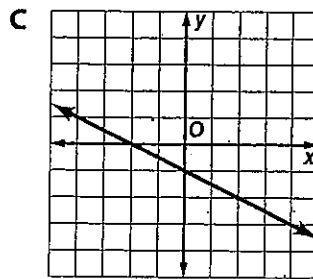
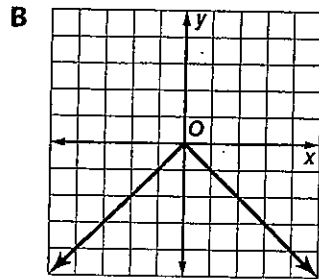
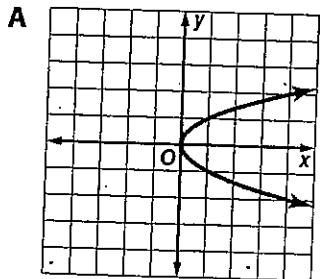
6. Which of the following relations is a function?

A	$x$	5	4	3	4
	$y$	4	4	2	2
B	$x$	0	2	1	0
	$y$	8	4	6	0
C	$x$	1	2	8	-4
	$y$	0	12	0	12
D	$x$	-1	-2	5	5
	$y$	-2	4	6	8

7. Which of the following relations is a function?



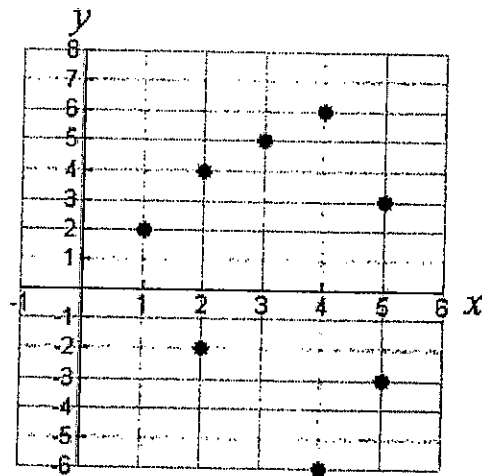
8. Which of the following relations is *not* a function?



9. The graph of a relation is shown at the right. Is this relation a function?

Choose:

- Yes
- No
- Cannot be determined from a graph



10. Is the relation depicted in the chart below a function?

X	-2	-1	0	1	2	3
Y	5	5	5	5	5	5

11. Given  $f(x) = 3x + 7$ , find  $f(5)$ .

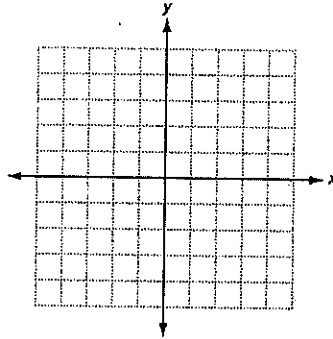
12. Given  $f(x) = 2x^2 - 3x + 6$ , find  $f(2.5)$ .

# Additional Practice

Express each relation as a table, as a graph, and as a mapping diagram.

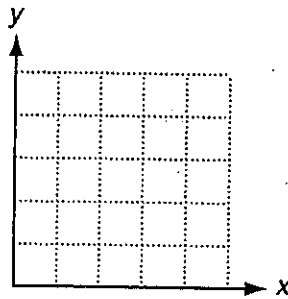
1.  $\{(-5, 3), (-2, 1), (1, -1), (4, -3)\}$

x	y



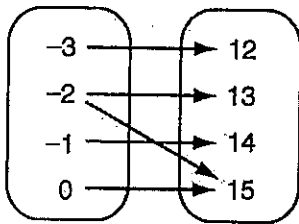
2.  $\{(4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5)\}$

x	y



Give the domain and range of each relation. Tell whether the relation is a function. Explain.

3.



D: \_\_\_\_\_

R: \_\_\_\_\_

Function? \_\_\_\_\_

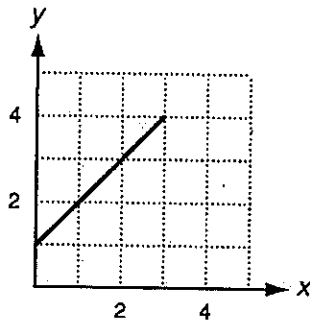
Explain: \_\_\_\_\_

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4.



D: \_\_\_\_\_

R: \_\_\_\_\_

Function? \_\_\_\_\_

Explain: \_\_\_\_\_

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5.

x	y
8	8
6	6
4	4
2	6
0	8

D: \_\_\_\_\_

R: \_\_\_\_\_

Function? \_\_\_\_\_

Explain: \_\_\_\_\_

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