Name Key
Date

How Do We Know If A Relation Is A Function?
How Do We Identify The Domain And Range Of A Function?

VOCABULARY

A **relation** is a set of ordered pairs.

The set of first coordinates from the ordered pairs is the **domain.** x-values

The set of second coordinates from the ordered pairs is the **range.** y-values

A **function** is a relation in which each domain value is paired with one range value. ← x-values DO NOT repeat

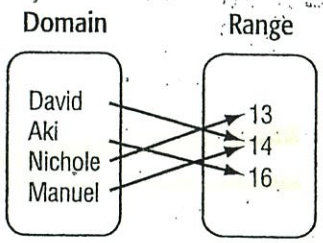
The **vertical line test** states that the graph of a relation is a function if every vertical line passes through no more than one point on the graph.

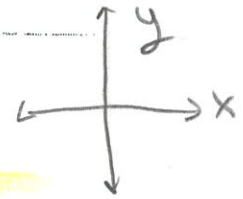
KEY CONCEPT: Identifying the Domain and Range of a Relation

Each element of the domain is paired with an element of the range: (Domain, Range).

{(David, 14), (Aki, 16), (Nichole, 13), (Manuel, 14)}

Domain: {David, Aki, Nichole, Manuel}
Range: {13, 14, 16}





An **ordered pair** names the location of a point on a coordinate grid. The x-coordinate (the first coordinate) tells you how far to the right or left of 0 the point is, and the y-coordinate tells you how far up or down from 0 the point is.

A **relation** is a set of ordered pairs, for example, $\{(-4,3), (4,9), (5,12)\}$.

The **domain** of a relation is the set of x-values of the relation. The **range** of a relation is the set of y-values of the relation. For the relation shown above, the domain is $\{-4, 4, 5\}$ and the range is $\{3, 9, 12\}$.

① Must be written in numerical order.
② Don't list repeats

A **function** is a relation in which each element of the domain corresponds to one and only one element of the range. In terms of ordered pairs, a function is a relation in which no two ordered pairs have the same x-coordinate. The relation above has no two x-coordinates that are the same, so that relation is a function.

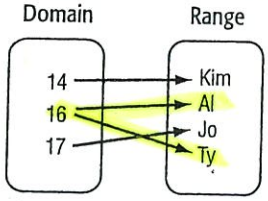
easy way.

the x-values don't repeat

A relation in which the first coordinate is never repeated is called a *function*.

Not a Function

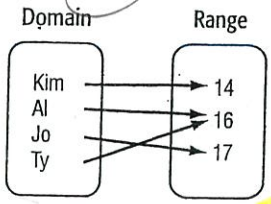
{(14, Kim), (16, Al), (17, Jo), (16, Ty)}



In a function, each domain value (age) can have only one range value (students). The domain value 16 has two range values: Al and Ty.

Function

{(Kim, 14), (Al, 16), (Jo, 17), (Ty, 16)}



If we switch the domain and range, the new relation is a function. Each domain value (student) has *exactly* one range value (age).

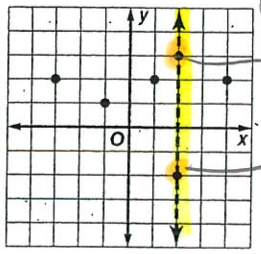
You can identify whether a given relation is a function by inspecting its ordered pairs.

Vertical Line Test

The graph of a relation on the coordinate plane can help you visually identify whether a relation is a function. In an ordered pair, the domain value is the x-coordinate and the range value is the y-coordinate. Because every point on a vertical line has the same x-coordinate, a vertical line can never be a function. The *vertical line test* states that a relation is a function if no vertical line passes through more than one point on the graph.

Use the vertical line test to identify whether each relation is a function.

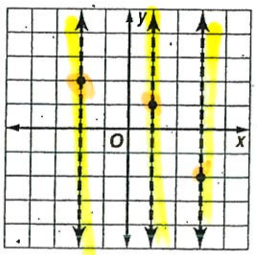
a.



This relation is *not* a function. At least one vertical line passes through two points.

Understanding the Solution Notice that (2, 3) and (2, –2) both have the same x-coordinate, 2.

b.



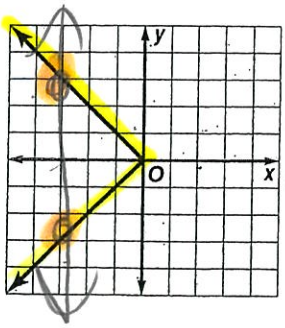
This relation is a function. No vertical line passes through more than one point on the graph.

Understanding the Solution No points have the same x-coordinate.

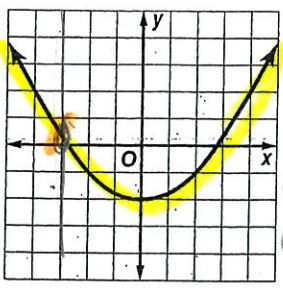
TRY IT!

Use the vertical line test to identify whether each relation is a function.

a.



b.



Functional Notation:

Traditionally, functions are referred to by the notation $f(x)$, but f need not be the only letter used in function names. Remember that $f(x)$ is telling you that the result will be "a function of x", or is dependent upon x. The statements $y = x^2$ and $f(x) = x^2$ are basically the same.

You may even see statements such as $f(x) = y = x^2$.

Example: A function is represented by $f(x) = 2x + 5$. Find $f(3)$.

To find $f(3)$, replace the x-value with 3. $f(\mathbf{3}) = 2(\mathbf{3}) + 5 = 11$.
The answer, 11, is called the image of 3 under $f(x)$.

Note: The $f(x)$ notation can be thought of as another way of representing the y-value, especially when graphing. The y-axis may even be labeled as the $f(x)$ axis.

Exercises

MULTIPLE CHOICE

1 What is the domain of the relation: {(0,1), (2, 1), (1, 3), (2, 4)}?

A {0, 1, 3}
B {1, 3, 4}
C {0, 1, 2}
D {0, 1, 2, 2}

2 What is the range of the function: {(6, 4), (3, 8), (7, 9), (9, 8)}?

A {4, 8, 9}
B {3, 4, 6, 8}
C {6, 7, 8, 9}
D {3, 6, 7, 9}

3 Which of the following relations is *not* a function?

A

x	1	2	3	4
y	2	4	6	8

B

x	0	2	4	6
y	2	4	8	10

C

x	1	2	3	4
y	2	2	2	2

D

x	1	2	4	4
y	8	6	4	2

4 Which of the following relations is a function?

A {(4, 6), (7, 1), (9, 5), (7, 5)}
B {(4, 6), (5, 1), (9, 5), (7, 5)}
C {(4, 6), (5, 1), (5, 9), (7, 5)}
D {(4, 6), (5, 1), (4, 3), (7, 5)}

5 Which of the following relations is *not* a function?

A {(−4, −3), (2, 0), (4, 3), (5, −4)}
B {(4, −3), (2, 0), (−4, 3), (−5, 4)}
C {(−4, −3), (−2, 0), (2, 3), (4, 3)}
D {(−4, −3), (2, 0), (4, 3), (2, −4)}

6 Which of the following relations is a function?

A

x	5	4	3	4
y	4	4	2	2

B

x	0	2	1	0
y	8	4	6	0

C

x	1	2	8	−4
y	0	12	0	12

D

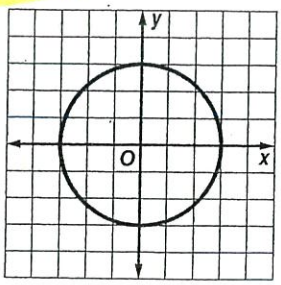
x	−1	−2	5	5
y	−2	4	6	8

7 Which of the following relations is a function?

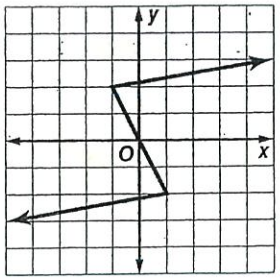
passes the V.L.T.

* vertical lines only goes through graph are time

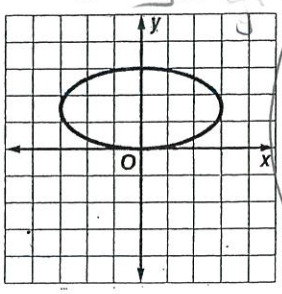
A



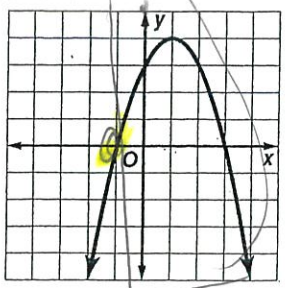
B



C



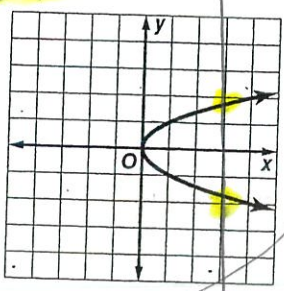
D



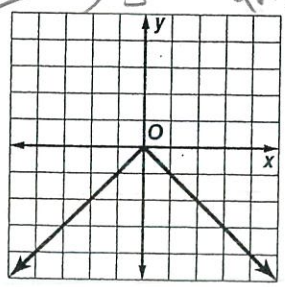
8 Which of the following relations is *not* a function?

fails vertical line goes through more than one time

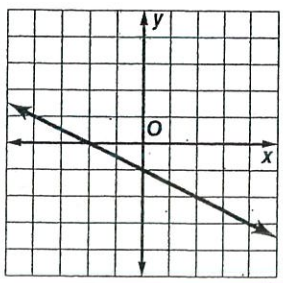
A



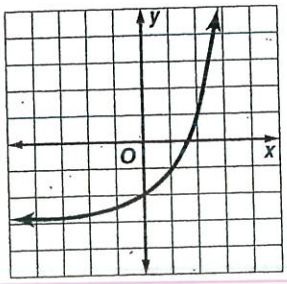
B



C



D



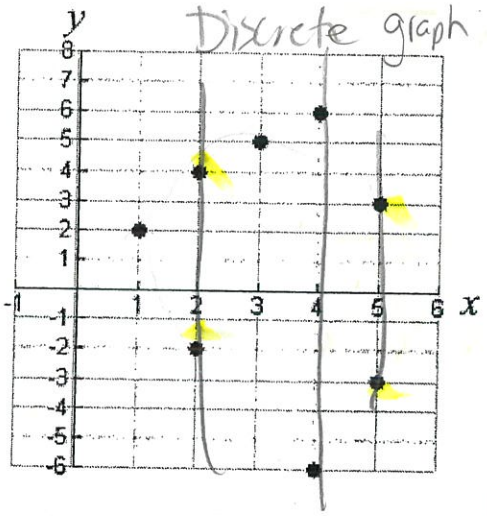
9. The graph of a relation is shown at the right. Is this relation a function?

Discrete graph

vertical line goes through more than 1 time

Choose:

- ○ Yes
- ○ No
- ○ Cannot be determined from a graph



10. Is the relation depicted in the chart below a function?

X	-2	-1	0	1	2	3
Y	5	5	5	5	5	5

Yes! It is a function b/c each element of the Domain corresponds to one and only one element of the range.

11. Given $f(x) = 3x + 7$, find $f(5)$.

$x = 5$

$f(5) = 3(5) + 7$

$f(5) = 15 + 7$

$f(5) = 22$

(5, 22)

12. Given $f(x) = 2x^2 - 3x + 6$, find $f(2.5)$.

$x = 2.5$

$f(2.5) = 2(2.5)^2 - 3(2.5) + 6$

$f(2.5) = 2(6.25) - 3(2.5) + 6$

$f(2.5) = 12.5 - 7.5 + 6$

$f(2.5) = 11$

(2.5, 11)

Name _____ Class _____ Date _____

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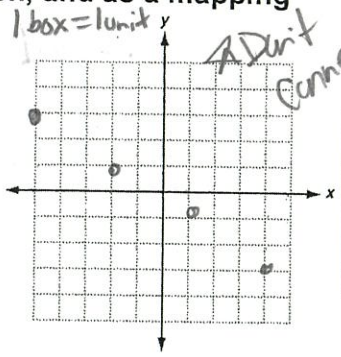
Additional Practice

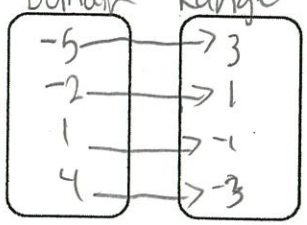
Express each relation as a table, as a graph, and as a mapping diagram.

1. {(−5, 3), (−2, 1), (1, −1), (4, −3)}

x	y
−5	3
−2	1
1	−1
4	−3

yes it is a function (x-values don't repeat)

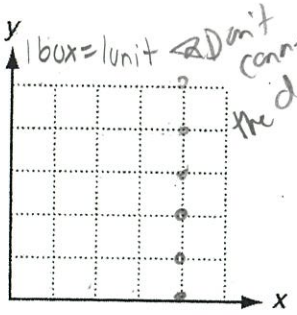


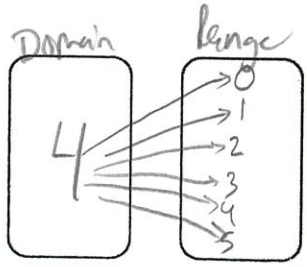


2. {(4, 0) (4, 1), (4, 2), (4, 3), (4, 4), (4, 5)}

x	y
4	0
4	1
4	2
4	3
4	4
4	5

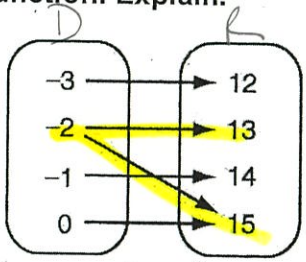
NOT a function (x-values repeat)





Give the domain and range of each relation. Tell whether the relation is a function. Explain.

3.



numerical order ✱ Don't list the repeats

(-2, 13) & (-2, 15)

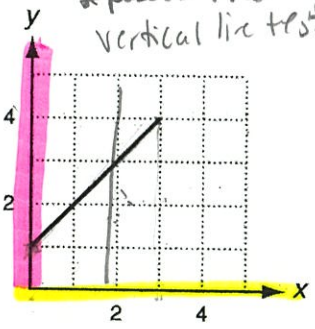
D: {-3, -2, -1, 0}

R: {12, 13, 14, 15}

Function? NO

Explain: B/C each element of the Domain does not correspond to one & only one element of the range

4.



✱ passes the vertical line test

D: {0 ≤ x ≤ 3}

R: {1 ≤ y ≤ 4}

Function? yes!

Explain: B/C each element of the domain corresponds to one and only one element of the range

5.

x	y
8	8
6	6
4	4
2	6
0	8

D R

look for crisscross

numerical order ✱ Don't list the repeats

D: {0, 2, 4, 6, 8}

R: {4, 6, 8}

Function? yes!

Explain: B/C each element of the domain corresponds to one & only one element of the range.