

Lesson 15: Solving Systems of Equations Graphically

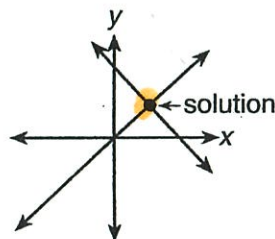
A **system of linear equations** consists of two or more linear equations. A **solution to a system of linear equations** is any ordered pair that is a solution to each equation in the system. In this lesson, you will solve systems of linear equations by graphing.

To solve a system of equations graphically, graph each of the equations in the system. The solution(s), if any, will be the ordered pair(s) of the point(s) of intersection of all the graphs.

There are three types of systems of linear equations. Each type has a different number of solutions. A **consistent system** has exactly one solution. An **inconsistent system** has no solution. A **dependent system** has an infinite number of solutions. You can determine the type of system by comparing the slopes and y-intercepts of the equations in the system.

Consistent System

- exactly one solution
- different slopes (same or diff y-int)

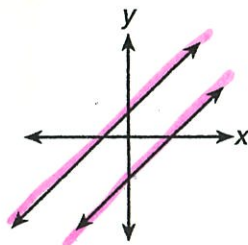


$x = 5$

$y = -x + 4$
 $y = 2x + 1$

Inconsistent System

- no solution
- same slope, different y-intercepts

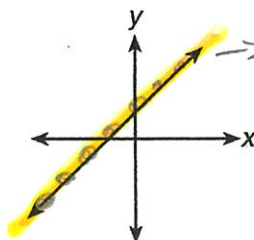


$5 \neq 7$

$y = 3x + 2$
 $y = 3x + 3$

Dependent System

- infinite number of solutions
- same slope, same y-intercept



→ 2 liters, 1 on top of the other

$5 = 5$

$y = 3x + 6$
 $y = 3x + 6$

For #'s 1-3: Without graphing, determine the type of system that is given and the number of solutions it has.

1) $x + 4y = 12$
 $3x + 12y = -24$

$$\begin{array}{r} x + 4y = 12 \\ -x \quad \quad x \\ \hline \end{array}$$

$$\frac{4y}{4} = \frac{-x + 12}{4}$$

$$y = -\frac{1}{4}x + 3$$

$$m = -\frac{1}{4} \leftarrow \text{same}$$

$$b = 3 \leftarrow \text{different}$$

$$\begin{array}{r} 3x + 12y = -24 \\ -3x \quad \quad -3x \\ \hline \end{array}$$

$$\frac{12y}{12} = \frac{-3x - 24}{12}$$

$$y = -\frac{1}{4}x - 2$$

$$m = -\frac{1}{4}$$

$$b = -2$$

Inconsistent
 No solutions

2) $4x - y = -9$
 $-12x + 3y = 27$

$$\begin{array}{r} 4x - y = -9 \\ -4x \quad \quad -4x \\ \hline \end{array}$$

$$\frac{-y}{-1} = \frac{-4x - 9}{-1}$$

$$y = 4x + 9$$

$$m = 4 \leftarrow \text{same}$$

$$b = 9 \leftarrow \text{same}$$

$$\begin{array}{r} -12x + 3y = 27 \\ +12x \quad \quad +12x \\ \hline \end{array}$$

$$\frac{3y}{3} = \frac{12x + 27}{3}$$

$$y = 4x + 9$$

$$m = 4$$

$$b = 9$$

Dependent
 Infinite

3) $2x - y = 4$
 $x + 4y = 20$

$$\begin{array}{r} 2x - y = 4 \\ -2x \quad \quad -2x \\ \hline \end{array}$$

$$\frac{-y}{-1} = \frac{-2x + 4}{-1}$$

$$y = 2x - 4$$

$$m = \frac{2}{1} \leftarrow \text{Different}$$

$$b = -4 \leftarrow \text{Different}$$

$$\begin{array}{r} x + 4y = 20 \\ -x \quad \quad -x \\ \hline \end{array}$$

$$\frac{4y}{4} = \frac{-x + 20}{4}$$

$$y = -\frac{1}{4}x + 5$$

$$m = -\frac{1}{4}$$

$$b = 5$$

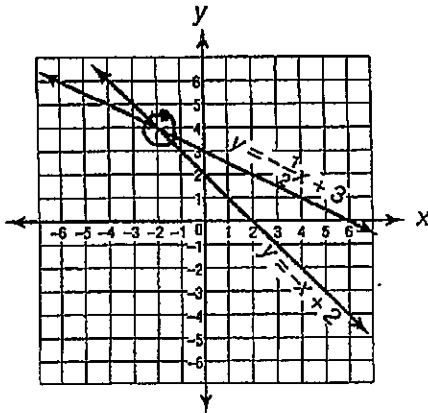
Consistent
 one solution
 (4, 4)

A check point
 $y = 4$ to check

Name _____

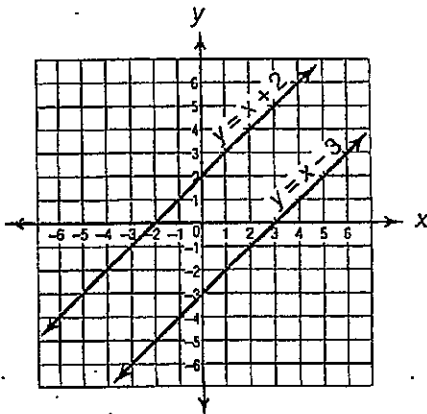
Systems Review

① What is the solution of the system of linear equations graphed below?



- A (4, -2)
- B (0, 2)
- C (0, 3)
- D (-2, 4)

② ~~Consistent~~
What is the solution of the system of linear equations graphed below?



- F all ordered pairs on both lines
- G There is no solution.
- H (0, 2)
- J (0, -3)

~~Consistent~~

They never intersect

③ Roland has to find the solution of this system of linear equations.

$$\frac{2x}{2} = \frac{4x-2}{2} \quad y = 2x - 1 \quad m: 2 \quad b: -1$$

$$\frac{3y}{3} = \frac{6x-3}{3} \quad y = 2x - 1 \quad m: 2 \quad b: -1$$

Without graphing, what is the solution?
[Hint: Divide both sides of the first equation by 2, and divide both sides of the second equation by 3. Then compare the equations.]

- A all ordered pairs on both lines
- B There is no solution.
- C (0, -2)
- D (0, -3)

~~Dependent~~

④ If two lines are parallel, what do you know about their equations?

- F They have all their solutions in common.
- G They have no solutions in common.
- H They have exactly one solution in common.
- J They have exactly two solutions in common.



~~Inconsistent~~

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What is the solution of this system of linear equations? Write your answer.

$$y = 3x + 2 \quad m: 3 \quad \frac{3}{1} \rightarrow B: 2$$

$$y = x \quad m: 1 \quad B: 0$$

Use the grid to sketch the graphs.

$$y = 3x + 2$$

$$m: \frac{3}{1}$$

$$B: 2$$

$$y = x$$

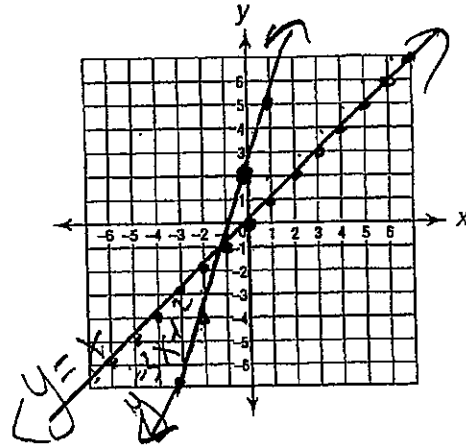
$$m: 1$$

$$B: 0$$

$(-1, -1)$

Answer: _____

Name: Consistent



Part 2: Inconsistent Question

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Part A

Graph this system of equations on the coordinate grid below.

$$y = \frac{1}{2}x + 4 \quad m: \frac{1}{2} \rightarrow B: 4$$

$$y = \frac{1}{2}x - 3 \quad m: \frac{1}{2} \rightarrow B: -3$$

$$y = \frac{1}{2}x + 4$$

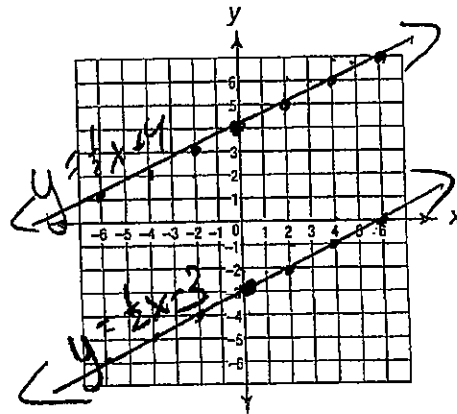
$$m: \frac{1}{2}$$

$$B: 4$$

$$y = \frac{1}{2}x - 3$$

$$m: \frac{1}{2}$$

$$B: -3$$



Part B

What is the solution of the system?

NO Solution

Part C

Name: Inconsistent

7 Part A: Graph the system of equations on the coordinate grid below.

$$y = x - 6 \quad m: \frac{1}{1} \quad B: -6$$

$$\frac{2y}{2} = \frac{2x - 12}{2}$$

$$y = x - 6 \quad m: \frac{1}{1} \quad B: -6$$

$$y = x - 6$$

$$m = \frac{1}{1}$$

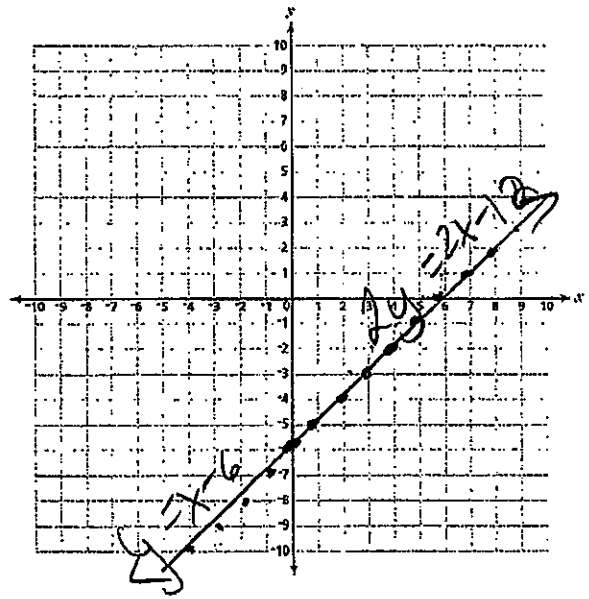
$$B = -6$$

$$\frac{2y}{2} = \frac{2x - 12}{2}$$

$$y = x - 6$$

$$m = \frac{1}{1}$$

$$B = -6$$



Part B: What is the solution of the system? Infinite # of solutions

Part C: What is the name of the type of system? Dependent

8 Part A: Graph the system of equations on the coordinate grid below.

$$y = \frac{1}{2}x - 4 \quad m: \frac{1}{2} \quad B: -4$$

$$\frac{2y}{2} = \frac{x + 6}{2}$$

$$y = \frac{1}{2}x + 3 \quad m: \frac{1}{2} \quad B: 3$$

$$y = \frac{1}{2}x - 4$$

$$m: \frac{1}{2}$$

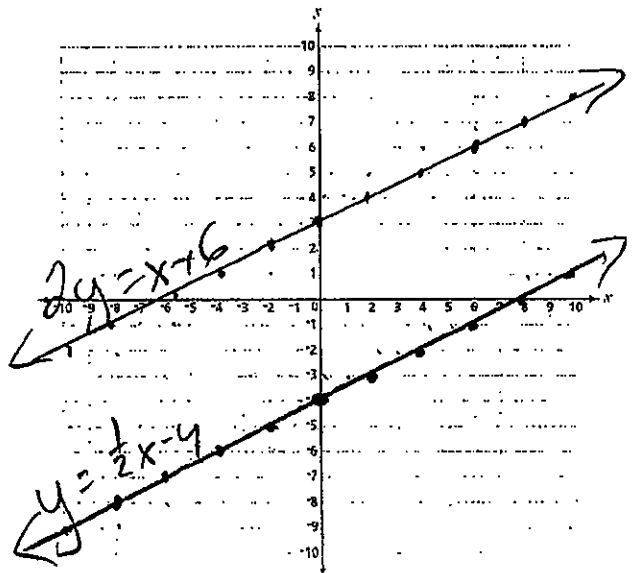
$$B: -4$$

$$\frac{2y}{2} = \frac{x + 6}{2}$$

$$y = \frac{1}{2}x + 3$$

$$m: \frac{1}{2}$$

$$B: 3$$



Part B: What is the solution of the system? No solutions

Part C: What is the name of the type of system? Inconsistent