

GOAL Represent piecewise functions**VOCABULARY**

Piecewise functions are represented by a combination of equations, each corresponding to a part of the domain.

A **step function** has a graph which resembles a set of stair steps. An example of a step function is the *greatest integer function*. This function is denoted by $g(x) = \llbracket x \rrbracket$, where for every real number x , $g(x)$ is the greatest integer less than or equal to x .

EXAMPLE 1 Evaluating a Piecewise Function

Evaluate $f(x)$ when (a) $x = -1$, (b) $x = 1$, and (c) $x = 3$.

$$f(x) = \begin{cases} 2x + 3, & \text{if } x < 0 \\ 2, & \text{if } 0 \leq x < 2 \\ -x + 1, & \text{if } x \geq 2 \end{cases}$$

SOLUTION

- a. $f(x) = 2x + 3$ Because $-1 < 0$, use first equation.
 $f(-1) = 2(-1) + 3 = 1$ Substitute -1 for x .
- b. $f(x) = 2$ Because $0 \leq 1 < 2$, use second equation.
 $f(1) = 2$ Substitute 1 for x .
- c. $f(x) = -x + 1$ Because $3 \geq 2$, use third equation.
 $f(3) = -3 + 1 = -2$ Substitute 3 for x .

Exercises for Example 1

Evaluate the function for the given value of x .

$$f(x) = \begin{cases} x + 1, & \text{if } x > 1 \\ -x - 2, & \text{if } x \leq 1 \end{cases}$$

$$g(x) = \begin{cases} 3x + 2, & \text{if } x < 5 \\ -2x, & \text{if } x \geq 5 \end{cases}$$

1. $g(5)$ 2. $f(0)$ 3. $f(3)$ 4. $g(-2)$

Practice with Examples

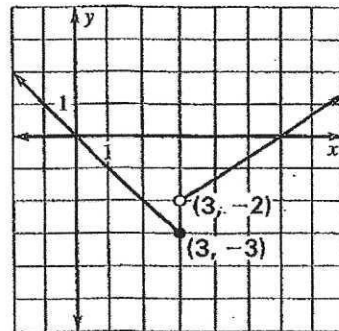
EXAMPLE 2 Graphing a Piecewise Function

Graph the function: $f(x) = \begin{cases} -x, & \text{if } x \leq 3 \\ \frac{2}{3}x - 4, & \text{if } x > 3 \end{cases}$

SOLUTION

To the right of $x = 3$, the graph is given by $y = \frac{2}{3}x - 4$. To the left of and including $x = 3$, the graph is given by $y = -x$.

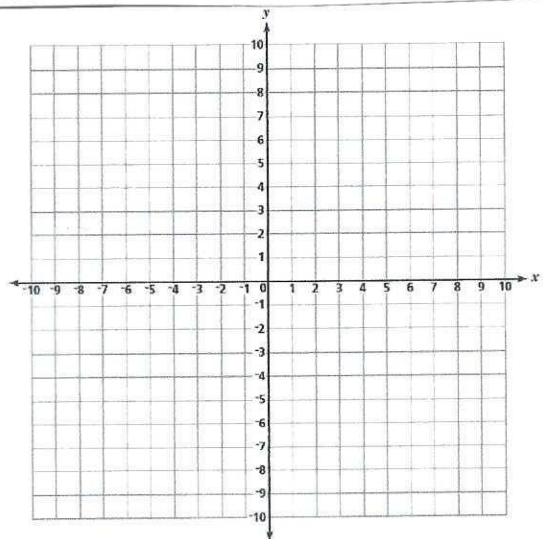
The graph consists of two rays.



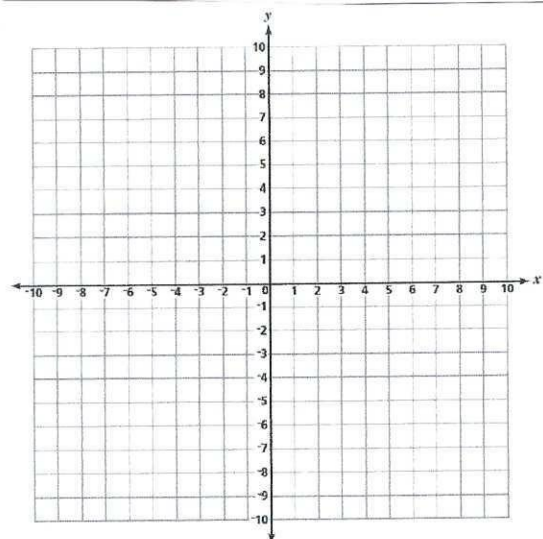
Exercises for Example 2

Graph the function.

5. $f(x) = \begin{cases} x + 2, & \text{if } x > 1 \\ -x + 2, & \text{if } x \leq 1 \end{cases}$



6. $f(x) = \begin{cases} \frac{1}{2}x + 4, & \text{if } x < 2 \\ -2x + 9, & \text{if } x \geq 2 \end{cases}$



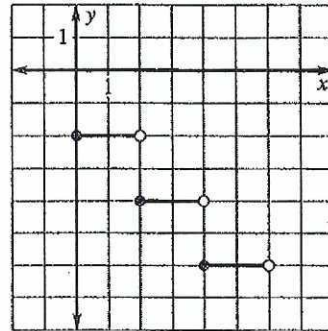
Practice with Examples

EXAMPLE 3 *Graphing a Step Function*

Graph the function. $f(x) = \begin{cases} -2, & \text{if } 0 \leq x < 2 \\ -4, & \text{if } 2 \leq x < 4 \\ -6, & \text{if } 4 \leq x < 6 \end{cases}$

SOLUTION

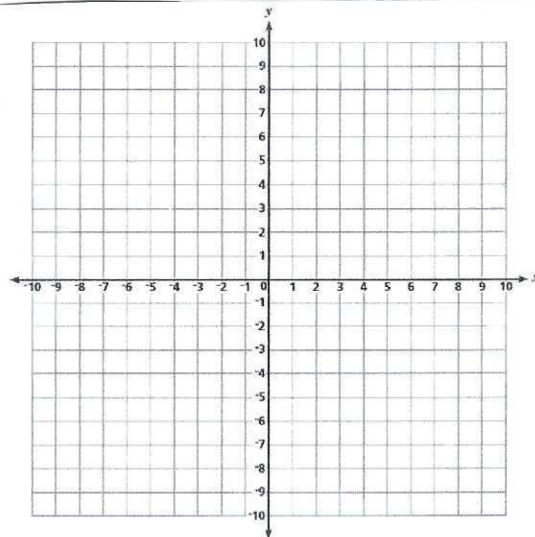
The graph is composed of three line segments, because the function has three parts. The intervals of x tell you that each line segment is 2 units in length and begins with a solid dot and ends with an open dot.



Exercises for Example 3

Graph the step function.

7. $f(x) = \begin{cases} 1, & \text{if } 0 < x \leq 1 \\ 3, & \text{if } 1 < x \leq 2 \\ 4, & \text{if } 2 < x \leq 3 \\ 6, & \text{if } 3 < x \leq 4 \end{cases}$



8. $f(x) = \begin{cases} -1, & \text{if } -2 \leq x < 1 \\ -2, & \text{if } 1 \leq x < 3 \\ -3, & \text{if } 3 \leq x < 6 \\ -4, & \text{if } 6 \leq x < 8 \end{cases}$

