Name Key

Mrs. Roumbos

Date:

8A Period

GOAL **Represent piecewise functions**

VOCABULARY

Piecewise functions are represented by a combination of equations, each corresponding to a part of the domain.

A **step function** has a graph which resembles a set of stair steps. An example of a step function is the *greatest integer function.* This function is denoted by $g(x) = [\![x]\!]$, where for every real number x, $g(x)$ is the greatest integer less than or equal to x.

EXAMPLE 1 *Evaluating a Piecewise Function*

Evaluate $f(x)$ when (a) $x = -1$, (b) $x = 1$, and (c) $x = 3$.

$$f(x) = \begin{cases} 2x + 3, & \text{if } x < 0 \\ 2, & \text{if } 0 \le x < 2 \\ -x + 1, & \text{if } x \ge 2 \end{cases}$$

SOLUTION

a. $f(x) = 2x + 3$ — Because $-1 < 0$, use first equation.

$f(-1) = 2(-1) + 3 = 1$ — Substitute -1 for x.

b. $f(x) = 2$ — Because $0 \le 1 < 2$, use second equation.

$f(1) = 2$ — Substitute 1 for x.

c. $f(x) = -x + 1$ — Because $3 \ge 2$, use third equation.

$f(3) = -3 + 1 = -2$ — Substitute 3 for x.

Exercises for Example 1

Evaluate the function for the given value of *x*.

$$f(x) = \begin{cases} x + 1, & \text{if } x > 1 \\ -x - 2, & \text{if } x \le 1 \end{cases} \qquad g(x) = \begin{cases} 3x + 2, & \text{if } x < 5 \\ -2x, & \text{if } x \ge 5 \end{cases}$$

1. $g(5)$

$g(x) = -2x$

$g(5) = -2(5)$

$g(5) = -10$

2. $f(0)$

$f(x) = -x - 2$

$f(0) = -(0) - 2$

$f(0) = 0 - 2$

$f(0) = -2$

3. $f(3)$

$f(x) = x + 1$

$f(3) = 3 + 1$

$f(3) = 4$

4. $g(-2)$

$g(x) = 3x + 2$

$g(-2) = 3(-2) + 2$

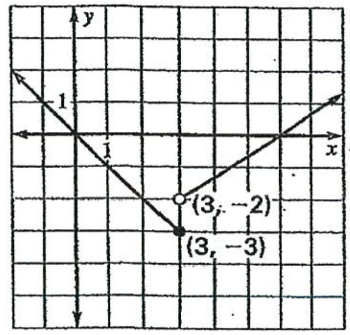
$g(-2) = -6 + 2$

$g(-2) = -4$

Practice with Examples

EXAMPLE 2 Graphing a Piecewise Function

Graph the function: $f(x) = \begin{cases} -x, & \text{if } x \le 3 \\ \frac{2}{3}x - 4, & \text{if } x > 3 \end{cases}$



SOLUTION

To the right of $x = 3$, the graph is given by $y = \frac{2}{3}x - 4$. To the left of and including $x = 3$, the graph is given by $y = -x$.

The graph consists of two rays.

Exercises for Example 2

Graph the function.

5. $f(x) = \begin{cases} x + 2, & \text{if } x > 1 \\ -x + 2, & \text{if } x \le 1 \end{cases}$

$f(x) = x + 2$, if $x > 1$ → open

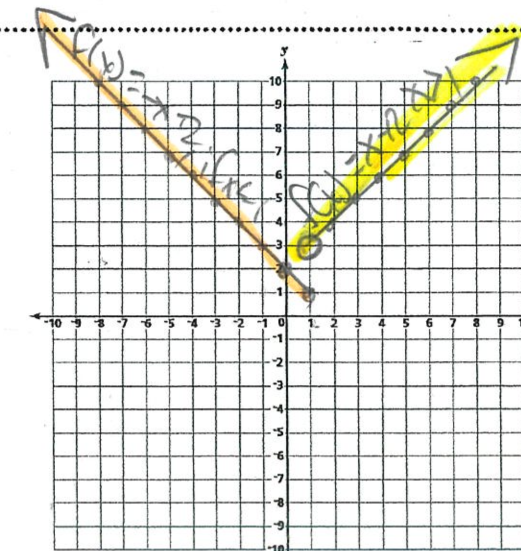
m: $\frac{1\uparrow}{1\rightarrow}$

B: 2

$f(x) = -x + 2$, if $x \le 1$ → closed

m: $\frac{-1\downarrow}{1\rightarrow}$

B: 2



$f(1) = 1$

$f(-4) = 6$

$f(17) = 19$

$f(x) = x + 2$

$f(17) = 17 + 2$

$f(17) = 19$

6. $f(x) = \begin{cases} \frac{1}{2}x + 4, & \text{if } x < 2 \\ -2x + 9, & \text{if } x \ge 2 \end{cases}$

$f(x) = \frac{1}{2}x + 4$, if $x < 2$ → open

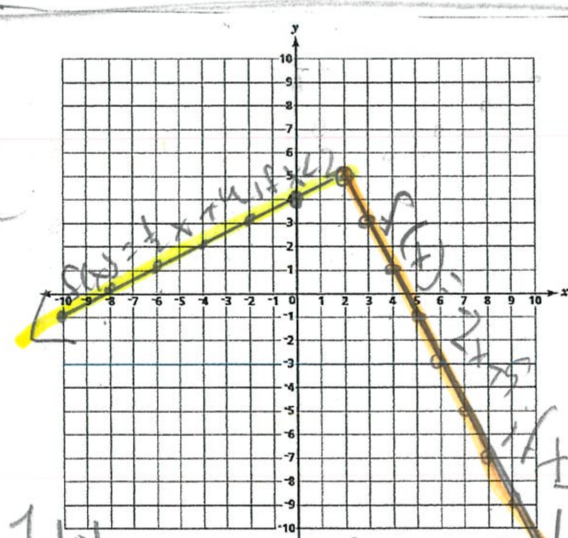
m: $\frac{1\uparrow}{2\rightarrow}$

B: 4

$f(x) = -2x + 9$, if $x \ge 2$ → closed

m: $\frac{-2\downarrow}{1\rightarrow}$

B: 9



★ If you have 1 dot open & 1 dot closed (when it is the same coordinates) make it closed

$f(2) = 5$

$f(-3) = 2.5$

$f(4) = 1$

$f(x) = \frac{1}{2}x + 4$

$f(-3) = \frac{1}{2}(-3) + 4$

$f(-3) = -1.5 + 4$

$f(-3) = 2.5$

Practice with Examples

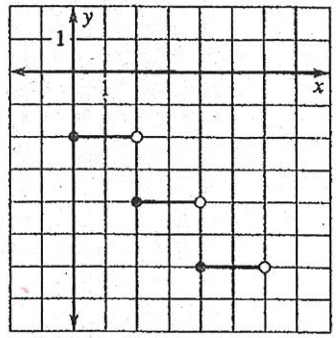
EXAMPLE 3 Graphing a Step Function

Graph the function. $f(x) = \begin{cases} -2, & \text{if } 0 \le x < 2 \\ -4, & \text{if } 2 \le x < 4 \\ -6, & \text{if } 4 \le x < 6 \end{cases}$

→ horizontal lines

SOLUTION

The graph is composed of three line segments, because the function has three parts. The intervals of x tell you that each line segment is 2 units in length and begins with a solid dot and ends with an open dot.



* Having a constant, will always result in a horizontal line.

↔ $y = \#$

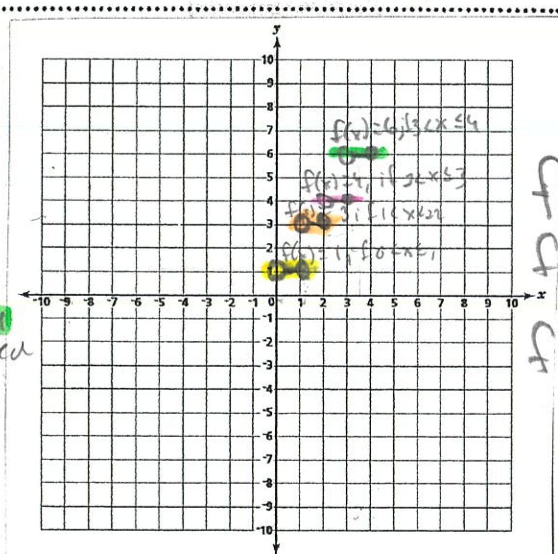
Exercises for Example 3

Graph the step function.

y-values

7. $f(x) = \begin{cases} 1, & \text{if } 0 < x \le 1 \\ 3, & \text{if } 1 < x \le 2 \\ 4, & \text{if } 2 < x \le 3 \\ 6, & \text{if } 3 < x \le 4 \end{cases}$

$f(x) = 1$, if $0 < x \le 1$ (open, closed)	$f(x) = 3$, if $1 < x \le 2$ (open, closed)	$f(x) = 4$, if $2 < x \le 3$ (open, closed)	$f(x) = 6$, if $3 < x \le 4$ (open, closed)
$m = 0$	$m = 0$	$m = 0$	$m = 0$
$B = 1$	$B = 3$	$B = 4$	$B = 6$



$f(1) = 1$

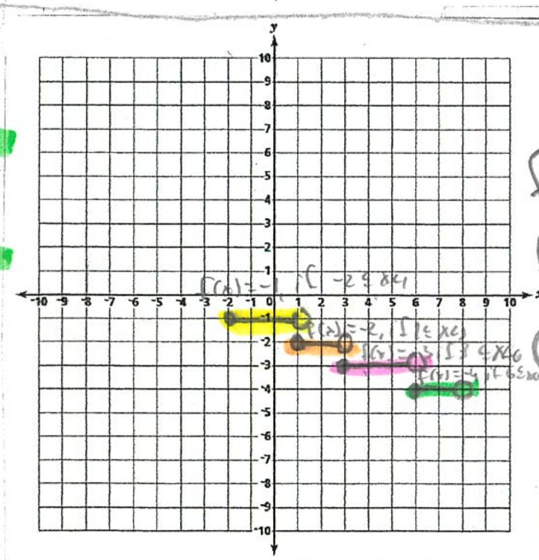
$f(3) = 4$

$f(3.5) = 6$

y-values

8. $f(x) = \begin{cases} -1, & \text{if } -2 \le x < 1 \\ -2, & \text{if } 1 \le x < 3 \\ -3, & \text{if } 3 \le x < 6 \\ -4, & \text{if } 6 \le x < 8 \end{cases}$

$f(x) = -1$, if $-2 \le x < 1$ (C, O)	$f(x) = -2$, if $1 \le x < 3$ (C, O)	$f(x) = -3$, if $3 \le x < 6$ (C, O)	$f(x) = -4$, if $6 \le x < 8$ (C, O)
$m =$ zero	$m =$ zero	$m =$ zero	$m =$ zero
$B = -1$	$B = -2$	$B = -3$	$B = -4$



$f(1) = -2$

$f(4) = -3$

$f(6) = -4$