Name ☆ Key

8A: Algebra 1

parent function $y = x^2$ vertex (0,0)

Date ____________

Period ____________

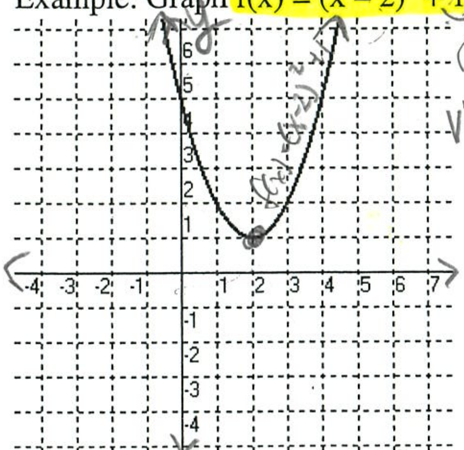
Graphing Parabolas in Vertex Form

$g(x) = a(x - h)^2 + k$, where (h,k) is the vertex

x y

I. Remember, when we're graphing a parabola, we want to find the vertex first, and then find two other points on either side of the vertex to graph so that we get the curved shape we're all familiar with. When a quadratic equation is in vertex form, the vertex is much easier to find than if the quadratic equation is in standard form.

- Example: Graph $f(x) = (x - 2)^2 + 1$ → Right 2 → 1 up (from (0,0)) V: (2,1)



The vertex is (2, 1). Since the x-value for the vertex is "2", then for the other x-values, we'll pick 3 and 4 on the left and 0 and 1 on the right. (The two nearest, nice x-values to x = 2).
Then, plug in 0, 1, 3, 4 for "x" and see what we get for f(x).

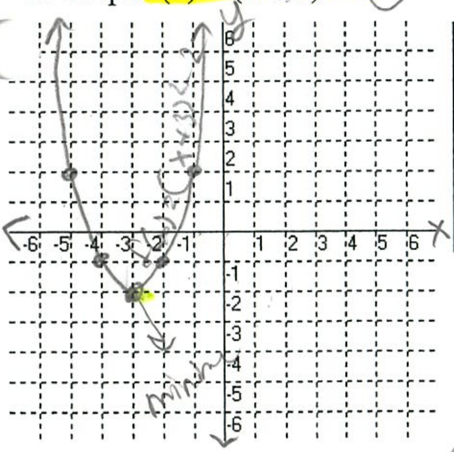
x	f(x)
0	5
1	2
2	1
3	2
4	5

← vertex!

Symmetric

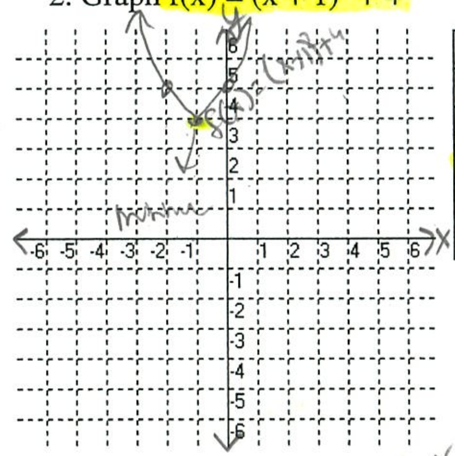
☆ arrows b/c constraints are not given

1. Graph $f(x) = (x + 3)^2 - 2$ → left 3 → 2 down from (0,0) Vertex = (-3, -2)



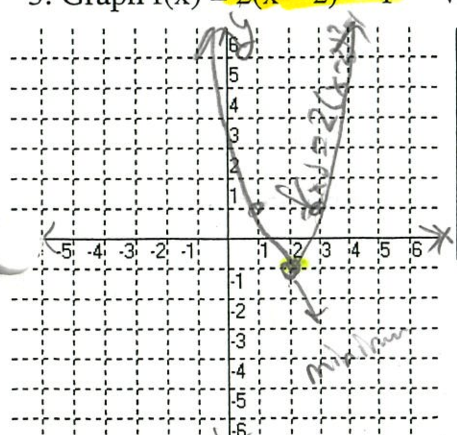
x	f(x)
-5	2
-4	-1
-3	-2
-2	-1
-1	2

2. Graph $f(x) = (x + 1)^2 + 4$ → left 1 → up 4 from (0,0) Vertex = (-1, 4)



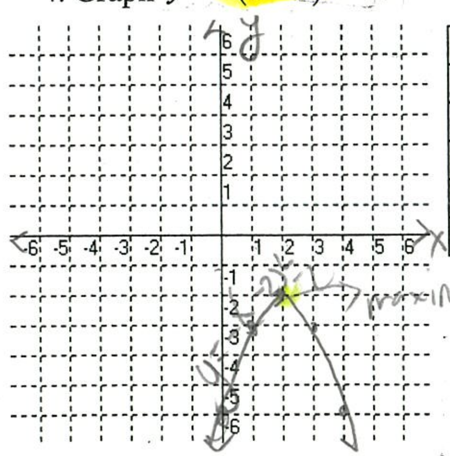
x	f(x)
-3	8
-2	5
-1	4
0	5
1	8

3. Graph $f(x) = 2(x - 2)^2 - 1$ → narrower by a scale factor of 2 → Right 2 → Down 1 from (0,0) Vertex = (2, -1)



x	f(x)
0	7
1	1
2	-1
3	1
4	7

4. Graph $y = -(x - 2)^2 - 2$ → Reflection over the x-axis → R+2 → Down 2 from (0,0) Vertex = (2, -2)



x	f(x)
0	-6
1	-3
2	-2
3	-3
4	-6

$g(x) = a(x-h)^2 + k \quad (h, k) = \text{vertex}$

x y opposite sign same sign

II. Without graphing, state the vertex for each of the following quadratic equations:

1) $y = (x - 5)^2 + 3$ opposite same $\boxed{V: (5, 3)}$

2) $y = (x + 3)^2 - 4$ $\boxed{V: (-3, -4)}$

3) $y = x^2 - 2.5$
$y = (x+0)^2 - 2.5$
$\boxed{V: (0, -2.5)}$

4) $y = (x + 4)^2 + 0$
$\boxed{V: (-4, 0)}$

III. Write a quadratic equation (in vertex form) whose graph will have the given vertex:

1) (1, -4) opp. same
$\boxed{y = (x-1)^2 - 4}$

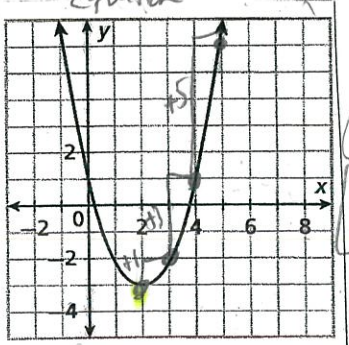
2) (-3, 5)
$\boxed{y = (x+3)^2 + 5}$

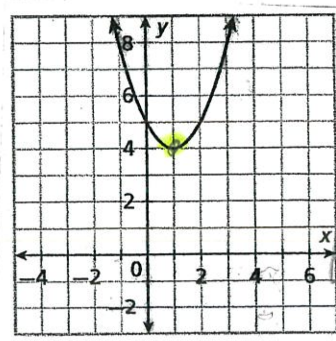
3) (0,100)
$\boxed{y = x^2 + 100}$

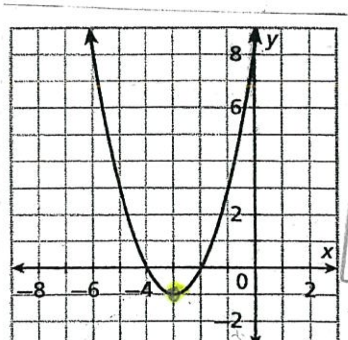
4) (-2,0)
$\boxed{y = (x+2)^2}$

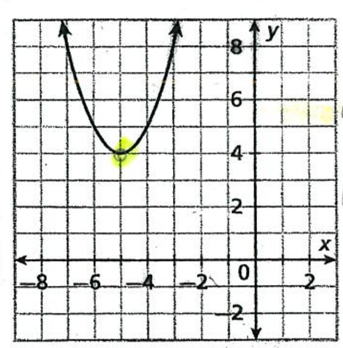
IV. Write a rule (in vertex form) for the quadratic function whose graph is shown below:

equation

1)

opp same
$\boxed{V: (2, -3)}$
$\boxed{f(x) = (x-2)^2 - 3}$

2)

$\boxed{V: (1, 4)}$
$\boxed{y = (x-1)^2 + 4}$

3)

$\boxed{V: (-3, -1)}$
$\boxed{y = (x+3)^2 - 1}$

4)

$\boxed{V: (-5, 4)}$
$\boxed{f(x) = (x+5)^2 + 4}$

V. Find the vertex and then write in vertex form. (algebraically)

1) $y = x^2 - 10x + 27$

$a=1 \quad b=-10 \quad c=27$

$x = \frac{-b}{2a}$

$x = \frac{-(-10)}{2(1)}$

$x = \frac{10}{2}$

$x = 5$

$y = x^2 - 10x + 27$

$y = (5)^2 - 10(5) + 27$

$y = 25 - 10(5) + 27$

$y = 25 - 50 + 27$

$y = 2$

$\boxed{V: (5,2)}$ $\boxed{f(x) = (x-5)^2 + 2}$

2) $y = x^2 - 12x + 40$

$a=1 \quad b=-12 \quad c=40$

$x = \frac{-b}{2a}$

$x = \frac{-(-12)}{2(1)}$

$x = \frac{12}{2}$

$x = 6$

$y = x^2 - 12x + 40$

$y = (6)^2 - 12(6) + 40$

$y = 36 - 12(6) + 40$

$y = 36 - 72 + 40$

$y = 4$

$\boxed{V: (6,4)}$ $\boxed{y = (x-6)^2 + 4}$

Review: Solve the following by **completing the square**: $x^2 + 8x - 9 = 0$

$ax^2 + bx = c$

$ax^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$

"a" must $= 1$

$b=8$

$x^2 + 8x - 9 = 0$

$+9 \quad +9$

$x^2 + 8x = 9$

$x^2 + 8x + \left(\frac{8}{2}\right)^2 = 9 + \left(\frac{8}{2}\right)^2$

$x^2 + 8x + 16 = 9 + 16$

$x^2 + 8x + 16 = 25$

$\frac{1}{2}$ of b

$\sqrt{(x+4)^2} = \sqrt{25}$

$x + 4 = \pm 5$

$x + 4 = 5$ | $x + 4 = -5$

$-4 \quad -4$ | $-4 \quad -4$

$\boxed{x = 1}$ | $\boxed{x = -9}$

$\{-9, 1\}$

VI. Write each quadratic in **vertex form** by **completing the square**. Then, identify the quadratics turning point. (vertex)

1) $y = x^2 - 8x + 11$

$b=-8$

$x^2 - 8x + \cancel{11} = y$

$\cancel{-11} \qquad -11$

$x^2 - 8x = y - 11$

$x^2 - 8x + \left(\frac{-8}{2}\right)^2 = y - 11 + \left(\frac{-8}{2}\right)^2$

$x^2 - 8x + 16 = y - 11 + 16$

$x^2 - 8x + 16 = y + 5$

$(x-4)^2 = y + \cancel{5}$

$-5 \qquad \cancel{-5}$

$\boxed{y = (x-4)^2 - 5}$

$\boxed{V: (4, -5)}$

2) $f(x) = x^2 + 6x - 2$

$b=6$

$x^2 + 6x \cancel{-2} = f(x)$

$\cancel{+2} \qquad +2$

$x^2 + 6x = f(x) + 2$

$x^2 + 6x + \left(\frac{6}{2}\right)^2 = f(x) + 2 + \left(\frac{6}{2}\right)^2$

$x^2 + 6x + 9 = f(x) + 2 + 9$

$x^2 + 6x + 9 = f(x) + 11$

$(x+3)^2 = f(x) + \cancel{11}$

$-11 \qquad \cancel{-11}$

$\boxed{f(x) = (x+3)^2 - 11}$

$\boxed{V: (-3, -11)}$

3) $f(x) = x^2 - 2x + 11$

$\boxed{b=-2}$

$$x^2 - 2x + \cancel{11} = f(x)$$
$$\cancel{-11} \qquad -11$$
$$x^2 - 2x = f(x) - 11$$
$$x^2 - 2x + \left(\frac{-2}{2}\right)^2 = f(x) - 11 + \left(\frac{-2}{2}\right)^2$$
$$x^2 - 2x + 1 = f(x) - 11 + 1$$
$$x^2 - 2x + 1 = f(x) - 10$$
$$(x-1)^2 = f(x) - \cancel{10}$$
$$+10 \qquad \cancel{+10}$$
$$\boxed{f(x) = (x-1)^2 + 10}$$
$$\boxed{V: (1, 10)}$$

4) $y = x^2 + 8x$

$\boxed{b=8}$

$$x^2 + 8x = y$$
$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = y + \left(\frac{8}{2}\right)^2$$
$$x^2 + 8x + 16 = y + 16$$
$$(x+4)^2 = y + \cancel{16}$$
$$-16 \qquad \cancel{-16}$$
$$\boxed{y = (x+4)^2 - 16}$$
$$\boxed{V: (-4, -16)}$$

****VII. Write the following equations in vertex form by <u>completing the square</u>.

1) $y = 5x^2 + 40x + 77$

$\boxed{b=8}$

$$\frac{5x^2}{5} + \frac{40x}{5} + \frac{77}{5} = \frac{y}{5}$$
$$x^2 + 8x + \cancel{\frac{77}{5}} = \frac{1}{5}y$$
$$-\cancel{\frac{77}{5}} \qquad -\frac{77}{5}$$
$$x^2 + 8x = \frac{1}{5}y - \frac{77}{5}$$
$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = \frac{1}{5}y - \frac{77}{5} + \left(\frac{8}{2}\right)^2$$
$$x^2 + 8x + 16 = \frac{1}{5}y - \frac{77}{5} + 16$$
$$x^2 + 8x + 16 = \frac{1}{5}y + \frac{3}{5}$$
$$(x+4)^2 = \frac{1}{5}y + \cancel{\frac{3}{5}}$$
$$-\frac{3}{5} \qquad -\cancel{\frac{3}{5}}$$

No Distribu Yes Distrib

$$5\left(\frac{1}{5}y\right) = 5\left((x+4)^2 - \frac{3}{5}\right)$$

* must have ( ) around the entire expression

$$\boxed{y = 5(x+4)^2 - 3}$$
$$\boxed{V: (-4, -3)}$$

2) $y = 2x^2 + 36x + 170$

$\boxed{b=18}$

$$\frac{2x^2}{2} + \frac{36x}{2} + \frac{170}{2} = \frac{y}{2}$$
$$x^2 + 18x + \cancel{85} = \frac{1}{2}y$$
$$\cancel{-85} \qquad -85$$
$$x^2 + 18x = \frac{1}{2}y - 85$$
$$x^2 + 18x + \left(\frac{18}{2}\right)^2 = \frac{1}{2}y - 85 + \left(\frac{18}{2}\right)^2$$
$$x^2 + 18x + 81 = \frac{1}{2}y - 85 + 81$$
$$x^2 + 18x + 81 = \frac{1}{2}y - 4$$
$$(x+9)^2 = \frac{1}{2}y - \cancel{4}$$
$$+4 \qquad \cancel{+4}$$

No Dist Yes Dist

$$2\left(\frac{1}{2}y\right) = 2\left((x+9)^2 + 4\right)$$

* must have ( ) around the entire expression

$$\boxed{y = 2(x+9)^2 + 8}$$
$$\boxed{V: (-9, 8)}$$