Name Key Date

8A; Algebra 1 Period

Verbal problems involving growth & decay

Many real world phenomena can be modeled by functions that describe how things grow or decay as time passes. Examples of such phenomena include the studies of populations, bacteria, the AIDS virus, radioactive substances, electricity, temperatures and credit payments, to mention a few.

Any quantity that grows or decays by a fixed percent at regular intervals is said to possess **exponential growth** or **exponential decay.**

At the Algebra level, there are two functions that can be easily used to illustrate the concepts of growth or decay in applied situations. When a quantity grows or decays by a fixed percent at regular intervals, the pattern can be represented by the functions,

☆ Must memorize

Growth: $A = P(1 + r)^n$	**Decay:** $A = P(1 - r)^n$

Principal (original) *P* = amount present before measuring growth/decay

☆ *r* = growth/decay rate (often a percent, but written as a decimal)

n = number of time intervals that have passed

A = after

Decay r = 50% when something is halved ($1 - ? = 50\%$) ($1 - .5 = .50$)

Growth r = 100% when something is doubled ($1 + ? = 200\%$) ($1 + 1 = 2.00$)

Examples:

1) In 2005, the population of a city was 25,000. The population increased by 20% in each of the next three years. If this rate of increase continues what will be the population of the city in 2012? (Don't consider a fractional part of a person.)

$A = P(1+r)^n$

$A = 25{,}000(1 + 20\%)^7$

$A = 25{,}000(1 + .20)^7$

$A = 25{,}000(1.2)^7$ → growth

$A = 89579.52$

2012 − 2005 = 7 ← n

Don't round

89,579 people

2) Daniel's Print Shop purchased a new printer for $35,000. Each year it depreciates (loses value) at a rate of 5%. What will its approximate value be at the end of the fourth year?

$A = P(1-r)^n$

$A = 35{,}000(1-5\%)^4$

$A = 35{,}000(1-.05)^4$

$A = \underbrace{35{,}000(.95)^4}_{}$ decay

$A = 28507.71875$

$28,507.72

$ is always rounded to the nearest hundredth

3) A bank account balance, *b*, for an account starting with *s* dollars, earning an annual interest rate, *r*, and left untouched for *n* years can be calculated as $b = s(1+r)^n$. Find a bank account balance to the nearest dollar, if the account starts with $100, has an annual rate of 4%, and the money left in the account for 12 years.

nearest whole #

$b = s(1+r)^n$

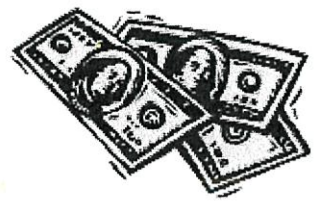
$b = 100(1+4\%)^{12}$

$b = 100(1+.04)^{12}$

$b = 100(1.04)^{12}$ growth

$b = 160.1032219$

$160



4) In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994? (Don't consider a fractional part of a person.)

Don't round

$A = P(1+r)^n$

$A = 285(1+75\%)^9$

$A = 285(1+.75)^9$

$A = 285(1.75)^9$ growth

$A = 43871.\cancel{98637}$

1994
−1985
9 ← N

43,871 cell phone subscribers

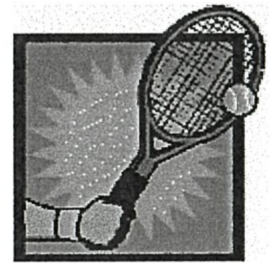
5) Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

P

N

50%

−



$A = P(1-r)^n$

$A = 128(1-50\%)^5$

$A = 128(1-.50)^5$

$A = 128(.50)^5 \rightarrow$ decays $\rightarrow y = \left(\frac{1}{2}\right)^x$

$A = 4$

4 players

6) A bank advertised a rate of 5% interest compounded annually on one of its CD's. If a 10 year old CD is now worth \$3,257.79, find its original price.

r

→ increases

→ N

A

P

$A = P(1+r)^n$

$3,257.79 = P(1+5\%)^{10}$

$3257.79 = P(1+.05)^{10}$

$\frac{3257.79}{(1.05)^{10}} = \frac{\boxed{P}(1.05)^{10}}{(1.05)^{10}}$

is always to the nearest hundredth

$P = 2000.000458$

\$2,000.00

7) The population of a town decreased at the rate of 2.5% per year. If the population in 2015 was 19,000 what was the population in 2000? (Don't consider a fractional part of a person)

−

r

A

$A = P(1-r)^n$

$19,000 = P(1-2.5\%)^{15}$

$19,000 = P(1-.025)^{15}$

$\frac{19,000}{(.975)^{15}} = \frac{P(.975)^{15}}{(.975)^{15}}$

$P = 27,776.93\cancel{775}$

27,776 people

2015
−2000
15 → N

Don't round

8) A bacteria population doubles every day. The starting population is 5.

100%

P

a) How large is the population after 5 days? → N

$A = P(1+r)^n$

$A = 5(1+100\%)^5$

$A = 5(1+1)^5$

$A = 5(2)^5$

$A = 160$

$y = 2^x$ growth/doubles

160 bacteria

b) How large is the population after two weeks? *must be in days

$A = P(1+r)^n$

$A = 5(1+100\%)^{14}$

2wks = 14 days

$A = 5(1+1)^{14}$

$A = 5(2)^{14}$ → growth/doubles

$A = 81,920$

81,920 bacteria

9) Hailey has begun a fitness program. The first week she ran 1 mile every day. Each week she increases the amount that she runs each day by 20%. In week 10, how many miles does she run each day? Give your answer to the nearest mile.

P

R

nearest whole #

10
− 1
9 → N

*Already counted week 1

$A = P(1+r)^n$

$A = 1(1+20\%)^9$

$A = 1(1+.20)^9$

$A = 1(1.2)^9$ → growth

$A = 5.159780352$

5 miles