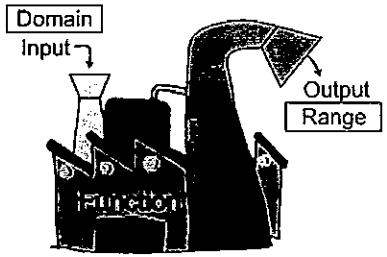
Name _____ Date _____

8A: Algebra 1 Period _____

Domain and Range



All of the values that can go into a relation or function (*input*) are called the domain.

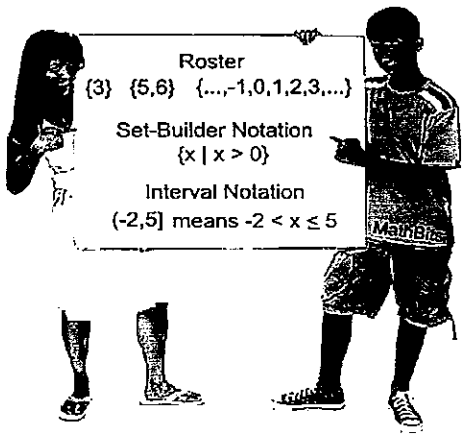
All of the values that come out of a relation or function (*output*) are called the range.
Range may also be referred to as "image".

Note that both relations and functions have domains and ranges.

The domain is the set of all first elements of ordered pairs (x-coordinates).
The range is the set of all second elements of ordered pairs (y-coordinates).
Only the elements "used" by the relation or function constitute the range.

Definition: Domain: all x-values that are to be used (independent values).
Range: all y-values that are used (dependent values).

Set Builder notation may be used to express domains and ranges.



Example 1:

State the domain and range of the following relation:
(eye color, student's name).
A = {(blue,Steve), (green,Elaine), (brown,Kyle), (blue,Marsha), (brown,Miranda), (green, Dylan)}
State whether the relation is a function.

Solution: Domain: {blue, green, brown}. Range: {Steve, Elaine, Marsha, Miranda, Dylan}.
No, this relation is not a function. The eye colors are repeated.

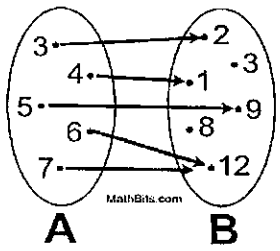
Example 2:

State the domain and range of the following relation: {(1,3), (-2,7), (3,-3), (4,5), (1,-3)}.
State whether the relation is a function.

Solution: Domain: {-2, 1, 3, 4}. Range: {-3, 3, 5, 7}.
While these listings appear in ascending order, ordering is not required. Do not, however, duplicate an element.
No, this relation is not a function. The x-value of "1" had two corresponding y-values (3 and -3).

Example 3:

State the domain and range for the elements matched in the diagram below.
State whether the matches form a function.



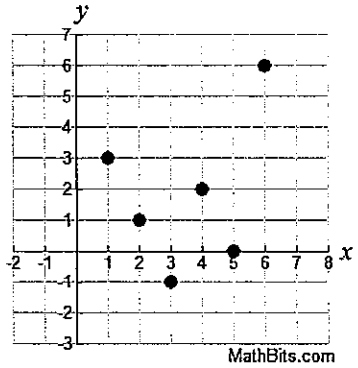
Solution: Domain: {3, 4, 5, 6, 7}. Range: {1, 2, 9, 12}.
Note that the range is only the elements that were used.

Yes, the relation {(3,2), (4,1), (5,9), (6,12), (7,12)}is a function.
No x-value repeats.

FYI: Set B = {1, 2, 3, 8, 9, 12} may be called the co-domain. It is the "possible" set from which output from the relation will fall. The co-domain is NOT necessarily the same as the range. There may be values in the co-domain that are never used.

Example 4:

State the domain and range associated with the scatter plot shown below.
State whether the scatter plot is a function.



Solution: Domain: {1, 2, 3, 4, 5, 6}.
(Be careful not to simply list the domain as $1 \le x \le 6$, which would imply ALL values between 1 and 6 inclusive, unless you specify "x is an integer".)

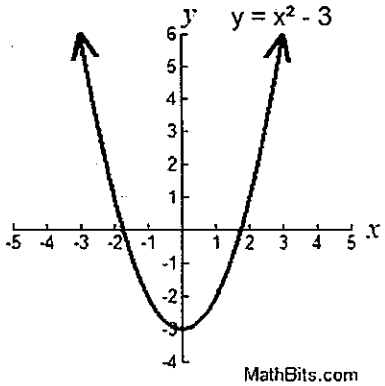
Range: { 0, -1, 1, 2, 3, 6}

Yes, this is a function. No x-values repeat, and it passes the Vertical Line Test for functions.

Note: Graphs that are composed of a series of dots, instead of a connected curve, are referred to as discrete graphs. A discrete domain is a set of input values that consist of only certain numbers in an interval.

Example 5:

State the domain and range associated with the graph below.
State whether this relation is a function.



Solution: Domain: $\mathbb{R}$ (all real numbers).
The arrows indicate that the graph continues off the visible grid, so assume that all real numbers are involved.

Range: $y \ge -3$ (may also be written as $\{y \in \mathbb{R} \mid y \ge -3\}$)

Yes, this relation is a function, since it passes the Vertical Line Test for functions.

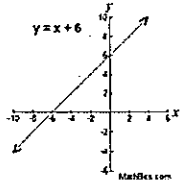
Note: Graphs that are composed of a connected curve are referred to as continuous graphs. A continuous domain is a set of input values that consists of all numbers in an interval.

Example 6:

State the appropriate domains for the functions shown below.
When functions are expressed as "rules" (formulas), be sure to think about possible "problem" spots before stating the domain. Remember that the domain is the x-values that are *allowed* by the function's equation. In most cases, a graph will help show the domain.

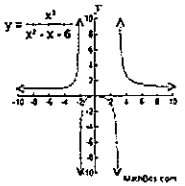
a) $y = x + 6$

Solution: This is the equation of a straight line. There are no "problem" spots with this straight line.
Domain: All real numbers, $\mathbb{R}$.



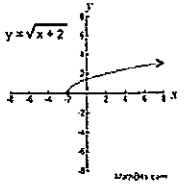
b) $y = \dfrac{x^2}{x^2 - x - 6}$

Solution: If a fractional expression contains a variable in its denominator, you need to check for division by zero. Set $x^2 - x - 6 = 0$ to find the problem spots of $x = -2$ and $x = 3$ which cause a zero denominator in this example..
Domain: $\mathbb{R}$ - {-2} - {3}.



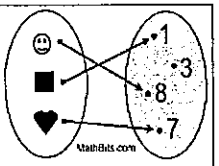
c) $y = \sqrt{x+2}$

Solution: The value under the radical needs to be 0 or a positive number (no negatives). To find the values that are OK, set $x + 2 \ge 0$, which is $x \ge -2$.
Domain: $x \ge -2$.



Practice Examples

1. Questions pertain to the function shown at the right.



a) What is the domain of the function?
Choose:
a) {1, 7, 8}
b) {☺, ■, ♥}
c) {1, 3, 7, 8}
d) {☺, ■, ♥, 1, 7, 8}

b) What is the range of the function?
Choose:
a) {1, 7, 8}
b) {☺, ■, ♥}
c) {1, 3, 7, 8}
d) {☺, ■, ♥, 1, 7, 8}

2. What is the domain of the function $f(x) = |x - 1|$?

Choose:

a) All Reals b) $[1, \infty)$ c) $(-\infty, 1)$ d) $(-\infty, -1)$

3. What is the range of the function $f(x) = x^2$?

Choose:

a) All Reals b) $(0, \infty)$ c) $[0, \infty)$ d) $(-\infty, 0]$

4. What is the domain of the function shown at the right?

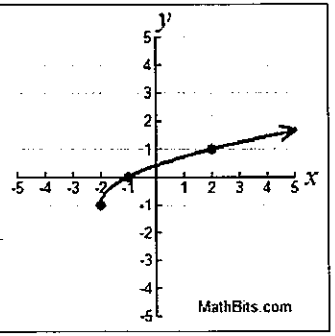
Choose:

a) $\{x \mid x > 2\}$

b) $\{x \mid x > -2\}$

c) $\{x \mid x \geq -2\}$

d) $\{x \mid x \leq -2\}$



5. What is the range of the relation $y = 2x^2 + 5x$, if the domain is the set $\{-2, 0, 1, 2\}$?

Choose:

a) $\{-18, 0, 7, 26\}$ c) $\{-18, 0, 8, 26\}$

b) $\{-2, 0, 8, 18\}$ d) $\{-2, 0, 7, 18\}$

6. A function is defined by the equation $y = -3x - 4$. If the domain is $1 \leq x \leq 5$, what is the minimum value in the range of the function?

Choose:

a) $-\infty$ b) -7 c) -10 d) -19

7. What is the domain of $f(x) = 2^x$?

Choose:

a) All integers c) All real numbers

b) $x \geq 0$ d) $x \leq 0$

8. In which function is the range equal to the domain?

Choose:

a) $y = 3^x$ c) $y = |x|$

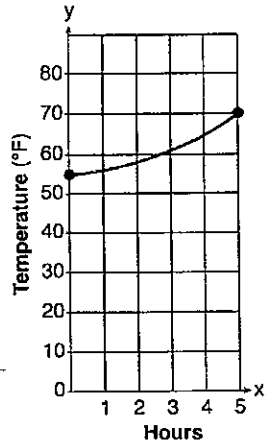
b) $y = x^2$ d) $y = x$

9. What is the domain of the function $f(x) = \sqrt{x}$?

Choose:

a) $x \geq 1$ b) $x \leq 1$ c) $x \geq 0$ d) $x \leq 0$

10. Air temperatures over a 5-hour period are shown in the graph at the right.



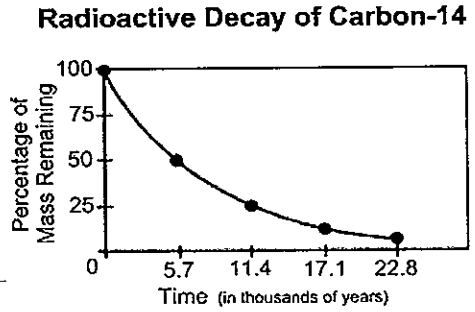
a) What is the domain of this set of data?
Choose:

a) $0 \le x \le 5$ c) $56 \le x \le 70$
b) $0 \le y \le 80$ d) $56 \le y \le 70$

b) What is the range of this set of data?
Choose:)

a) $0 \le x \le 5$ c) $56 \le x \le 70$
b) $0 \le y \le 80$ d) $56 \le y \le 70$

11. Questions pertain to the graph at the right.



a) Which type of function could be used to model this data?
Choose:

a) linear
b) quadratic
c) exponential

b) Which choice represents the domain of the data as displayed in the graph?
Choose:

a) [0, 100] b) [5700, 22800] c) [0, 22800] d) [25,100]

c) Which choice represents the range of the data as displayed in the graph?
Choose:

a) [0,100] b) [5700, 22800] c) [0, 22800] d) [25,100]

12. The domain of $f(x) = 3x + 2$ is $\{-1 \le x \le 4\}$.
Which integer is **not** in the range?

Choose:

a) -5 b) -1 c) 0 d) 11