

slope only for a line  $\rightarrow$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$f(x) = y$$

## Average Rate of Change

Directions: Find the average rate of change for the following problems. Show your work.

The **average rate of change** of a function  $y = f(x)$  over the interval  $[x_1, x_2]$  is  $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

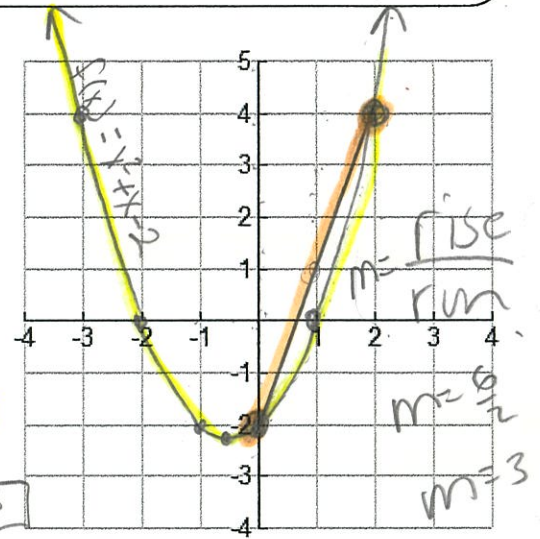
The average rate of change of a function corresponds to the slope of the line segment (called the secant line) connecting the two endpoints of the given interval.

x	f(x)
-3	4
-2	0
-1	-2
-0.5	-2.25
0	-2
1	0
2	4

1. a. Graph:  $f(x) = x^2 + x - 2$ .  
 b. Plot the points (2,4) and (0,-2) on the graph.  
 c. Draw a line segment connecting these two points.  
 d. What is the slope of this line segment?  $\frac{6}{2} = 3$   
 e. Using the formula for "average rate of change", find the average rate of change on the interval from  $x = 0$  to  $x = 2$ .

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(0)}{2 - 0} = \frac{4 - (-2)}{2 - 0} = \frac{6}{2} = 3$$

they value when  $x=2$



2. Given:

x	-2	-1	0	1	2
y	-8	-1	0	1	8

- a. Determine the average rate of change of  $y$  over the interval from  $x = -1$  to  $x = 1$ .  
 $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{1 - (-1)}{1 - (-1)} = \frac{2}{2} = 1$   
 b. Determine the average rate of change of  $y$  over the interval  $-2 \leq x \leq 1$ .  
 $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{1 - (-8)}{1 - (-2)} = \frac{9}{3} = 3$   
 c. Determine the average rate of change of  $y$  over the interval  $[-2, 2]$ .  
 $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{8 - (-8)}{2 - (-2)} = \frac{16}{4} = 4$

3. Calculate the average rate of change of the function over the given interval.

$f(x) = 4x + 3$  over the interval  $[2, 5]$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(5) - f(2)}{5 - 2} = \frac{23 - 11}{5 - 2} = \frac{12}{3} = 4$$

f(x)	x
11	2
15	3
19	4
23	5

$f(x) = 4x + 3$   
 $f(5) = 4(5) + 3 = 20 + 3 = 23$   
 $f(2) = 4(2) + 3 = 8 + 3 = 11$

4. Calculate the average rate of change in library income between 2010 and 2013.

Year	Library Income in Dollars
2008	14,587
2009	15,678
2010	16,988
2011	18,389
2012	19,089
2013	20,870

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{f(2013) - f(2010)}{2013 - 2010}$$

$$\frac{20,870 - 16,988}{2013 - 2010}$$

$$\frac{3882}{3} = 1294$$

\$1294/year

5. Find the average rate of change of the function shown that represents the amount of money in a savings account in Lender's Bank?

Week	Balance
1	\$128
2	\$142
3	\$156
4	\$170
5	\$184

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{f(2) - f(1)}{2 - 1}$$

$$\frac{142 - 128}{2 - 1} = \frac{14}{1} = 14$$

\$14/week

6. When the average rate of change of a function is constant, the function is linear. Determine if the following function is linear by examining the average rates of change.

x	-3	-1	0	4
f(x)	6	2	0	-8

$$\frac{\Delta y}{\Delta x}$$

$$\frac{-4}{2} = -2 \quad \frac{-2}{1} = -2 \quad \frac{-8}{4} = -2$$

It is linear b/c the rates of change are constant

7. For the following tables, decide if they are linear, quadratic, cubic, or neither.

K.O.L. is constant → 1st try  
 → 2nd try → 3rd try  
 shortcut:  $x$   $x^2$   $x^3$   
 only works when the Domain increases by 1

NOT constant at all (excl. abs. value)

a)  $x$   $y$

Temp	Miles
-1	1
0	3
1	5
2	7
3	9
4	11

b)

x	y
-2	-7
-1	0
0	1
1	2
2	9
3	28

+1  
+1  
+1  
+1  
+1  
+1

+2  
+2  
+2  
+2  
+2

+1  
+1  
+1  
+1  
+1

+7  
+1  
+1  
+7  
+19

+6  
+0  
+6  
+12

Linear B/c the rate of change is constant on the 1st try or it has a 1st common difference

Cubic B/c the rate of change is constant on the 3rd try or it has 3rd common difference

c)

x	y
-3	8
-2	3
-1	0
0	-1
1	0

d)

x	y
-3	3
-2	2
-1	1
0	0
1	1

+1  
+1  
+1  
+1

-5  
-3  
-1  
+1

+2  
+2  
+2

+1  
+1  
+1  
+1

-1  
-1  
-1  
+1

+0  
+0  
+2

Quadratic B/c the rate of change is constant on the 2nd try or it has a 2nd common difference

Neither B/c the rate of change is never constant

8. Joseph conducted a science experiment involving the growth of bacteria. He measures the number of bacteria hourly for 6 hours. The data is summarized in the accompanying table. What type of regression would best fit the data?

- a) Linear
- b) Exponential
- c) Quadratic
- d) Absolute Value

Y-values are Doubled, tripled, halved etc.

Hours	Number of Bacteria
1	100
2	200
3	400
4	800
5	1600

+1  
+1  
+1  
+1

+100  
+200  
+400  
+800

+100  
+200  
+400

Given  $f(x) = 2x^2$ . Find a value  $b$  such that the average rate of change of  $f(x)$  from  $x = 1$  to  $x = b$  equals 12.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = 12$$

$$\frac{f(b) - f(1)}{b - 1} = 12$$

$$\frac{2b^2 - 2}{b - 1} = 12$$

$$2b^2 - 2 = 12(b - 1)$$

$$2b^2 - 2 = 12b - 12$$

$$-12b + 12 - 12b + 12$$

$$2b^2 - 12b + 10 = 0$$

$$2(b^2 - 6b + 5) = 0$$

$$2(b - 5)(b - 1) = 0$$

2 ≠ 0	$b - 5 = 0$	$b - 1 = 0$
	$+5 + 5$	$+1 + 1$
	$b = 5$	$b = 1$

$$b = 5$$

$$f(x) = 2x^2$$

$$f(1) = 2(1)^2$$

$$f(1) = 2(1)$$

$$f(1) = 2$$


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$$f(x) = 2x^2$$

$$f(b) = 2b^2$$

Calculator Check

for 3rd page problem

STAT 1: Edit (that will get you  $L_1 + L_2$ )

put #'s into  $L_1 + L_2$  (press enter after each #)

To turn on points: 2nd

y=) 1) cr + 

Zoom 9: STAT to

See graph