

● End Behavior

Definition:

End behavior refers to the appearance of a graph as it is followed farther and farther in either direction (as the x -value gets very large or very small).

We have seen graphs with an arrow on either (or both) ends of the function indicating that the function continues "in the same manner" as the x -values continue to increase or decrease.

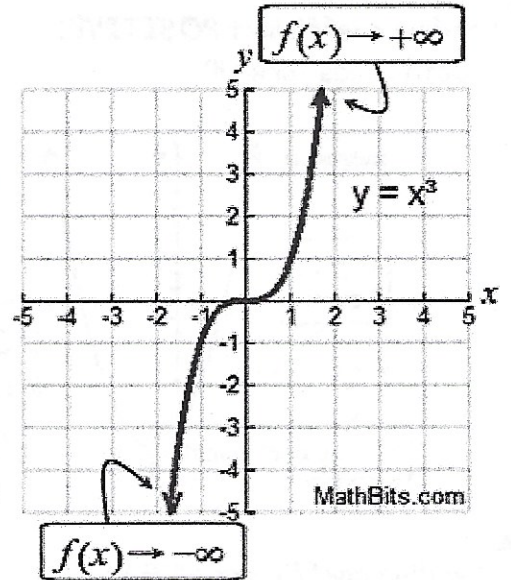
In the graph at the right, the arrow on the right end of the function tells us that as the x -values get larger and larger, the function (the y -values) will also get larger and larger. We say: "as x approaches positive infinity, $f(x)$ approaches positive infinity." And we

write: $f(x) \rightarrow +\infty$, as $x \rightarrow +\infty$

$f(x) = y$

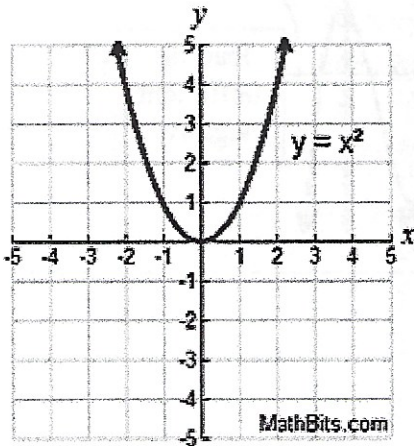
The arrow on the left end of this function says, as the x -values get smaller and smaller, the function will get smaller and smaller. We say: "as x approaches negative infinity, $f(x)$ approaches negative infinity."

And we write: $f(x) \rightarrow -\infty$, as $x \rightarrow -\infty$



The arrow (\rightarrow) is read "approaches"

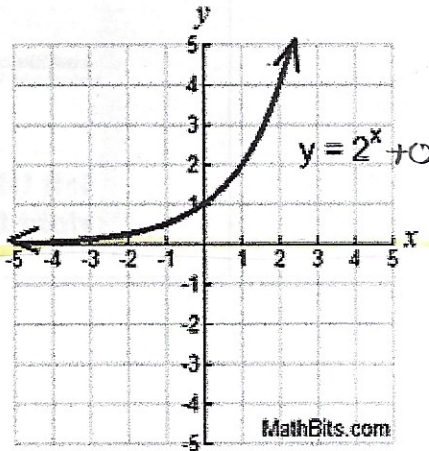
Other possibilities:



Both ends approaching positive infinity.

$f(x) \rightarrow +\infty$, as $x \rightarrow +\infty$

$f(x) \rightarrow +\infty$, as $x \rightarrow -\infty$

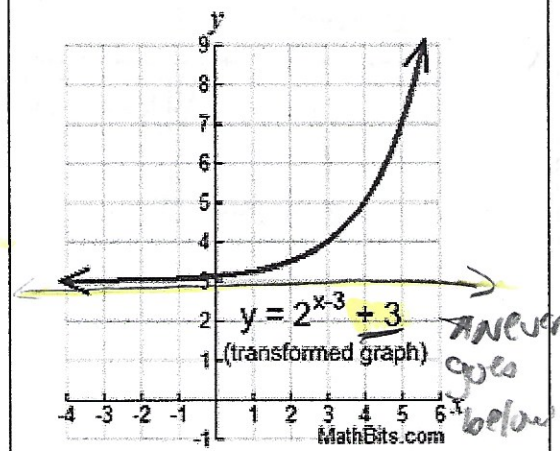


Right end approaching + infinity.

Left end approaching zero.

$f(x) \rightarrow +\infty$, as $x \rightarrow +\infty$

$f(x) \rightarrow 0$, as $x \rightarrow -\infty$



Right end approaching + infinity.

Left end approaching three.

$f(x) \rightarrow +\infty$, as $x \rightarrow +\infty$

$f(x) \rightarrow 3$, as $x \rightarrow -\infty$

Special End Behavior Information for Polynomial Functions:

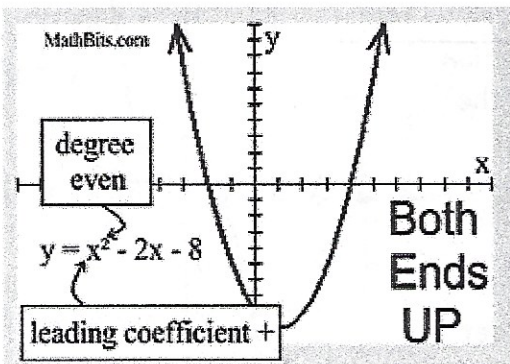
Polynomial functions have some special characteristics regarding end behavior, based upon whether the DEGREE of the polynomial is an odd or even number, and whether the SIGN of the leading coefficient is positive or negative. You can predict the end behavior of these functions without looking at the graph.

Even Degree Polynomial

("ends" behave similar to a quadratic)

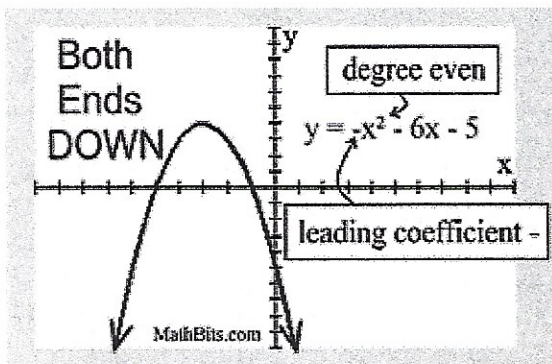
Leading coefficient POSITIVE:

Both "ends" are UP.



Leading coefficient NEGATIVE:

Both "ends" are DOWN.

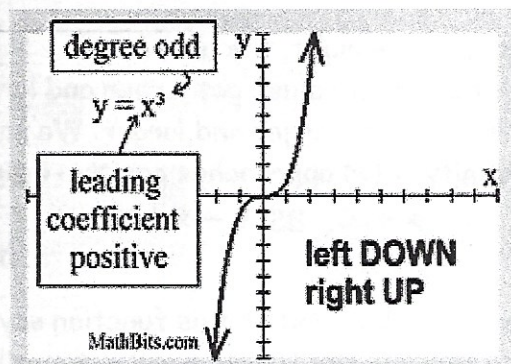


Odd Degree Polynomial

("ends" behave similar to a cubic)

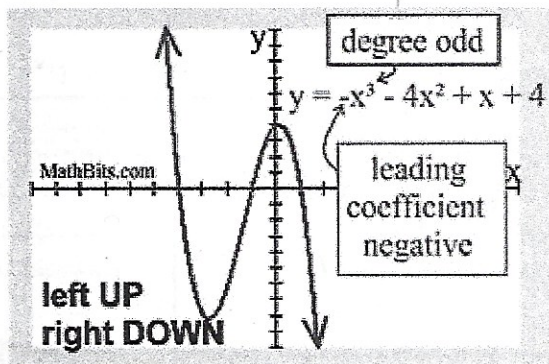
Leading coefficient POSITIVE:

Left end is DOWN and right end is UP.



Leading coefficient NEGATIVE:

Left end is UP and right end is DOWN.



1. Regarding the graph at the right:

a) Which of the following points is a relative maximum for this function?

Choose:

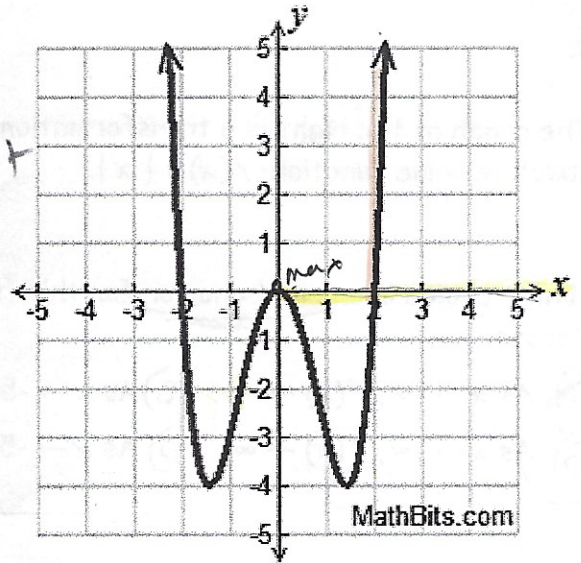
A) (-1.4, -4)

B) (1.4, -4)

C) (0, 0)

D) (2, 0)

has to be a turning point



b) Which choice describes the end behavior as x approaches infinity? $+\infty$

Choose:

A) $f(x) \rightarrow -\infty$

C) $f(x) \rightarrow 2$

B) $f(x) \rightarrow \infty$

D) $f(x) \rightarrow 3$

approach

2.

Regarding the graph at the right:

$$f(x) = 2\sqrt{x+1} - 2$$

a) On which interval is the function positive? $y > 0$, above the x-axis

Choose: Describe w/ x -values

A) $[-2, \infty)$ B) $(0, \infty)$ C) $[-1, \infty)$ D) $[-1, 0)$

b) On which interval is the function negative? $y < 0$ + below the x-axis

Choose: Describe w/ x -values

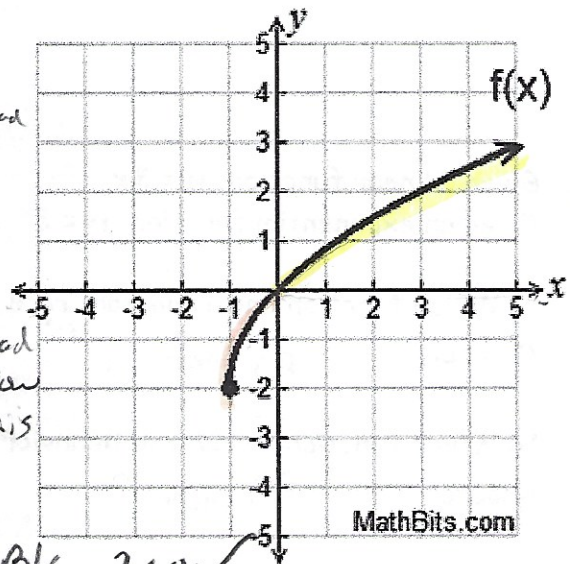
A) $[-2, \infty)$ B) $[-1, \infty)$ C) $[-2, 0]$ D) $[-1, 0)$

c) Which choice is an end behavior for this function?

Choose:

~~A) As $x \rightarrow \infty$, $f(x) \rightarrow 3$.~~ ~~B) As $x \rightarrow -1$, $f(x) \rightarrow -1$.~~

~~C) As $x \rightarrow \infty$, $f(x) \rightarrow -2$.~~ D) As $x \rightarrow -1$, $f(x) \rightarrow -2$.



I + II Quad

$y > 0$, above the x-axis

III + IV Quad

$y < 0$ + below the x-axis

I, B/C $-2 < 0$ ✓

$$f(x) = y$$

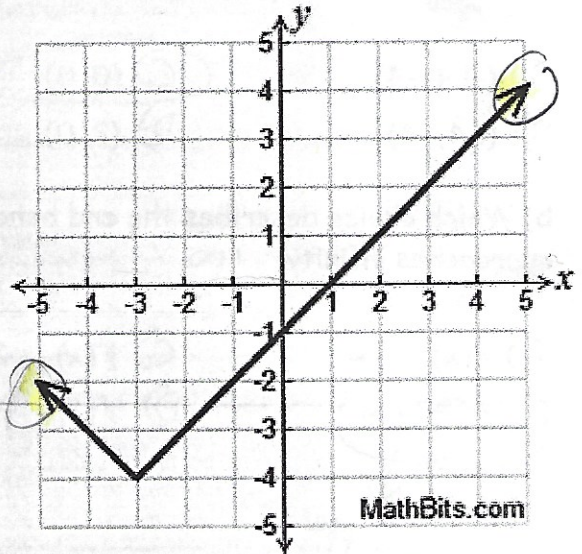
3.

The graph at the right is a transformation of the absolute value function, $f(x) = |x|$.

Which choice is an end behavior for this function?

Choose:

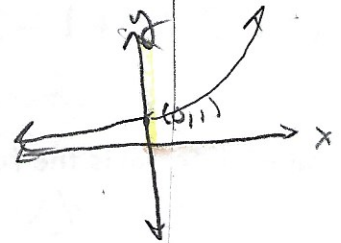
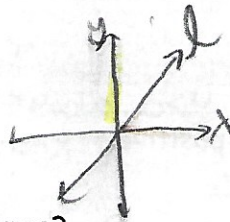
- ~~A) As $x \rightarrow -\infty, f(x) \rightarrow -\infty$.~~ C) As $x \rightarrow -5, f(x) \rightarrow \infty$.
- B) As $x \rightarrow -\infty, f(x) \rightarrow \infty$.** D) As $x \rightarrow -5, f(x) \rightarrow -\infty$.



4.

Given a linear function: $y_1 = 3x$

Given an exponential function: $y_2 = 3^x$



a) Which function has a constant rate of change?

Choose:

A) linear

B) exponential

C) both

b) Which function has an end behavior $y \rightarrow +\infty$ as $x \rightarrow +\infty$?

Choose:

A) linear

B) exponential

C) both

5.

Consider the graph of $y = 4^x$.

a) What happens to y as x gets smaller? $-\infty$

Choose:

A) y approaches 1

C) y approaches 2

B) y approaches 0

D) y approaches 4

NEVER = 0

