Name ☆ Key ☆
Date

Patterns and Sequences

A **sequence** is a set of numbers that follows a pattern. When you know the pattern, you can find more numbers in the sequence. Each number in a sequence is called a **term**.

Example 1:

Write a rule to describe the sequence 3, 7, 11, 15, …. Then **find the next three terms** in the sequence.

3, 7, 11, 15, 19, 23, 27 . . .
+4 +4 +4 +4 +4 +4

The rule for this sequence is Start with 3 and add by 4 repeatedly.
same #

aka linear function
The sequence in Example 1 is called an arithmetic sequence. Each term of an **arithmetic sequence** is found by **adding** a fixed number (called the **common difference**) to the previous term. ☆ If you can't figure out the difference: subtract the 2nd term by the 1st term ex 7-3=4

d = difference

What is the common difference in Example 1? d=4

The common difference in a sequence may also be negative. For example, in the sequence 25, 22, 19, 16, 13, 10, 7, …, the rule is *Start with* 25 *and add* -3 *repeatedly.*
-3 -3 -3 -3 -3 -3 d=

Identify the common difference in each arithmetic sequence. Then find the next three terms.

a) 3, 4, 5, 6, 7, 8, 9, 10
+1 +1 +1 +1 +1 +1 +1
common difference = 1

b) 6, 4, 2, 0, -2, -4, -6, -8
-2 -2 -2 -2 -2 -2 -2
common difference = -2

c) 9, $8\frac{1}{2}$, 8, $7\frac{1}{2}$, 7, $6\frac{1}{2}$, 6
$-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$
common difference = $-\frac{1}{2}$

Example 2:

On the first day of school 4 students came to extra help. On the second day 12 students came, and in the next few days 36, 108, and 324 students came to extra help. Write a rule to describe this sequence. Then find the next three terms.

4, 12, 36, 108, 324, 972, 2916, 8748
·3 ·3 ·3 ·3 ·3 ·3 ·3

Rule: Start with 4 and multiply by 3 repeatedly.

r=3
ratio

* If you can't figure out ratio: Divide 2nd term by 1st term ex $12 \div 4 = 3$

The sequence in example 2 is a geometric sequence. Each term of a **geometric sequence** is found by **multiplying** the previous term by a fixed number (called the **common ratio**).

aka exponential function

r = ratio

same #

Identify the common ratio in each geometric sequence.

a) 1, 3, 9, 27, … $\cdot 3$ $\cdot 3$ $\cdot 3$ $r = 3$

b) 100, 10, 1, $\frac{1}{10}$, … $r = \frac{1}{10}$

c) 5, 10, 20, 40, … $\cdot 2$ $r = 2$

Practice: Write a rule to describe each sequence. Then find the next three terms in the sequence.

Geom. a) 3, 6, 12, 24, 48, 96, 192 Rule: Start with 3 & multiply by 2 repeatedly $r=2$

Arith. b) 24, 21, 18, 15, 12, 9, 6 Rule: Start with 24 + add by −3 repeatedly $d=-3$

Geom c) 625, 125, 25, 5, 1, $\frac{1}{5}$ Rule: Start with 625 + multiply by $\frac{1}{5}$ repeatedly $r=\frac{1}{5}$

Not every sequence is arithmetic or geometric. You can also use an algebraic expression to describe the terms of many different sequences.

Example:

linear x^1

exponential $y=2^x$

can get values from calc $y=$ + the table

Evaluate for n = 1, 2, 3, 4, and 5

Tell whether the sequence formed is *arithmetic*, *geometric*, or *neither*.

n	f(n)
1	4
2	7
3	12
4	19
5	28

1) $n^2 + 3$ Neither (Quadratic: x^2)

n=1 n=2 n=3 n=4 n=5

4, 7, 12, 19, 28 → can get #s in calc (y=/table)

+3 +5 +7 +9

+2 +2 +2

2nd common difference

2) $5n + 8$ Arithmetic (linear: x^1)

n=1 n=2 n=3 n=4 n=5

13, 18, 23, 28, 33

+5 +5 +5 +5 → 1st common difference

n	f(n)
1	13
2	18
3	23
4	28
5	33

Fibonacci Sequence

In the Fibonacci sequence 1, 1, 2, 3, 5, 8, …, you can find each term (after the first two terms) by adding certain terms together. Find the pattern and use it to write the next three terms of the sequence.

1 1 2 3 5 8 13, 21, 34, …

* Add the 2 previous terms together to get the next term

Fill in the following blanks by using the information you learned from today's Do Now and today's lesson.

**An arithmetic sequence results in a linear function. d = difference

**A geometric sequence results in a exponential function. r = ratio

Name ~~key~~

More Intro to Sequences

DO NOW

Find the next three terms in each sequence.

1) 1, -3, 9, -27, 81, ...

$\cdot -3 \quad \cdot -3 \quad \cdot -3 \quad \cdot -3$

$r = -3$ Geometric

$\boxed{-243, 729, -2187}$

2) 14, 34, 54, 74, 94, ...

+20 +20 +20 +20

$d = 20$ Arithmetic

$\boxed{114, 134, 154}$

3) 1, 4, 9, 16, 25, 36, ...

perfect squares

Quadratic

+3 +5 +7 +9 +11 +13

+2 +2 +2 +2 +2

$\boxed{49, 64, 81}$

2nd common difference

A **sequence** is an ordered list. It is a function whose domain is the natural numbers {1, 2, 3, 4, 5, ...}.

term #'s

No decimals, fractions, Neg. #'s or 0!

Sequence:	**1,**	**5,**	**9,**	**13,**	**17,**	**21,**	**...**
Notation for terms of the sequence:	a_1	a_2	a_3	a_4	a_5	a_6	

Information about Sequences:

- Each number in a sequence is called a term.
- Terms are referenced in a subscripted form, where the natural number subscripts, {1, 2, 3, ...}, refer to the location (position) of the term in the sequence. The first term is denoted a_1 or $a(1)$, the second term a_2 or $a(2)$, and so on. The n^{th} term is a_n or $a(n)$. a sub n, a of n
  a sub 1, a of 1. a sub 2, a of 2
- The terms in a sequence may or may not have a pattern.
- A subscripted form of a sequence is represented by $a_1, a_2, a_3, \ldots a_n, \ldots$.
- A functional form of a sequence is represented by $f(1), f(2), f(3), \ldots, f(n), \ldots$.
- Sequences **are** functions.

Sequences can be expressed in various forms:

Term Number	Term	Subscript Notation	Function Notation
1	1	a_1	$f(1)$
2	5	a_2	$f(2)$
3	9	a_3	$f(3)$
4	13	a_4	$f(4)$
5	17	a_5	$f(5)$
6	21	a_6	$f(6)$
n	⋮	a_n	$f(n)$

{1, 5, 9, 13, 17, 21, ...}(list)

Subscripted notation:

$a_n = 4n - 3$ (explicit form)

$a_1 = 1;\ a_n = a_{n-1} + 4$ (recursive form)

Functional notation:

$f(n) = 4n - 3$ (explicit form)

$f(1) = 1; f(n) = f(n-1) + 4$ (recursive form)

Note: Not all functions can be defined by an explicit and/or recursive formula.

A sequence may appear as an explicit formula. An ***explicit formula*** designates the n^{th} term of the sequence, a_n, as an expression of n (where n = the terms location).

Example: {1, 5, 9, 13, 17, 21, ...} can be written $a_n = 4n - 3$.
(a formula in terms of n)

A sequence may appear as a recursive formula. A **recursive formula** designates the starting term, a_1, and the n^{th} term of the sequence, a_n, as an expression containing the previous term (the term before it), a_{n-1}.

Example: {1, 5, 9, 13, 17, 21, ...} can be written $a_1 = 1;\ a_n = a_{n-1} + 4$.
(two-part formula in terms of the preceding term)

Example 1

1st term n=1, 2nd ter n=2, 3rd ter n=3 etc

Consider the sequence below. If we represent this sequence with the letter *a* then do the following.

4, 8, 16, 32, 64, 128, 256

a) Find a(3) — n=3 → the third term value (y-value when x or n =3) $\boxed{16}$

c) Find a_2 → 2nd term value $\boxed{8}$

e) Find $a_5 - a_4$ — $64 - 32 = \boxed{32}$

b) Find a(1) + a(7) — $4 + 256 = \boxed{260}$

d) Find $(a_1)^2$ — $(4)^2 = \boxed{16}$

f) Solve for n: a(n) = 128 — $\boxed{n=6}$ → what term gives you value of 128

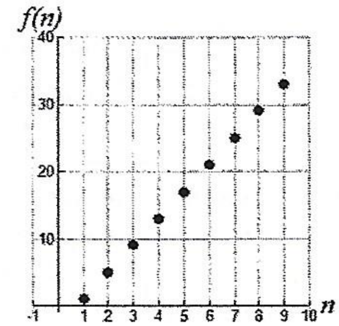
B/c a(6)=128

ARITHMETIC SEQUENCE

If a sequence adds a fixed amount from one term to the next, it is referred to as an arithmetic sequence. The number added to each term is constant (always the same) and is called the *common difference, d.* The graph of an arithmetic sequence will be a **linear function.**

Sequence: {1, 5, 9, 13, 17, 21, 25, 29, ...}

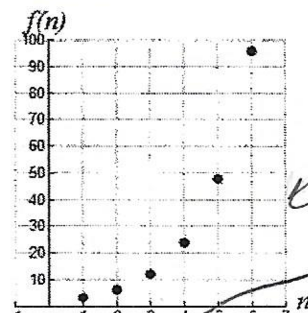
n	1	2	3	4	5	6	7	8
$f(n)$	1	5	9	13	17	21	25	29



GEOMETRIC SEQUENCE

If a sequence multiplies a fixed amount from one term to the next, it is referred to as a geometric sequence. The number multiplied is constant (always the same) and is called the *common ratio, r.* The graph of a geometric sequence will be an **exponential function.**

Sequence: {3, 6, 12, 24, 48, 96, ...}



Why don't we connect the points?

Discrete graph

It's only those specific terms it's not the #'s in between

x. Term #'s / Inputs / Domain : natural #'s only (No decimals or Negative #'s)

y Answers/Output/Range : may be decimals or negatives