Name Key
8A; Algebra 1

Date ____________
Period____________

Explicit Formula for Arithmetic Sequences

We already discussed that sequences can be expressed in various forms.

Explicit Forms:
Certain sequences (not all) can be defined (expressed) as an "explicit" formula. An explicit formula will create a sequence using *n*, the number location of each term. If you can find an explicit formula for a sequence, you will be able to quickly and easily find any term in the sequence simply by replacing *n* with the number of the term you seek.

An *explicit formula* designates the n^{th} term of the sequence, as an expression of n (where n = the term's location). It defines the sequence as a formula in terms of n. It may be written in either subscript notation a_n, or in functional notation, $f(n)$.

"a sub n"   "f of n"

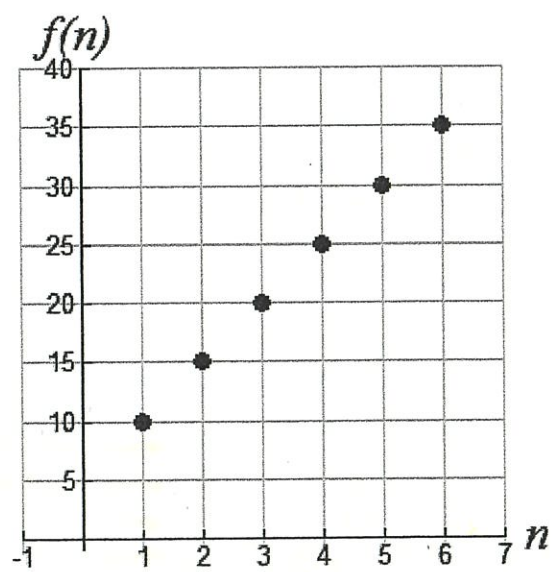
Example 1: Sequence: {10, 15, 20, 25, 30, 35, ...}. Find an explicit formula.

This example is an arithmetic sequence(the same number, 5, is added to each term to get to the next term).

Term Number	Term Value	Subscript Notation	Function Notation
1	**10**	a_1	$f(1)$
2	**15**	a_2	$f(2)$
3	**20**	a_3	$f(3)$
4	**25**	a_4	$f(4)$
5	**30**	a_5	$f(5)$
6	**35**	a_6	$f(6)$
n	⋮	a_n	$f(n)$

Explicit Formula:
in subscript notation: $a_n = 5n + 5$
in function notation: $f(n) = 5n + 5$

* Dots are not connected can't have partial term



MathBits.com

This sequence is graphed in the first quadrant. Remember that the domain consists of the natural numbers, {1, 2, 3, ...}, and the range consists of the terms of the sequence. Notice that this sequence has a linear appearance. The rate of change between each of the points is "5 over 1". While the n value increases by a constant value of one, the $f(n)$ value increases by a constant value of 5 (for this graph).

Will arithmetic sequences be linear functions?

$y = 5x + 5$ — m, b

slope "d"   y-int a_0 ↑ B/c that is where x=0 (where you start)

yes!

$y = mx + b$ — slope, y-int

It is easy to see that the explicit formula works once you are given the formula. Unfortunately, it is not always easy to come up with explicit formulas, when all you have is a list of the terms.

If your sequence is arithmetic, it will help if you look at the pattern of what is happening in the sequence.

n	1	2	3	4	5	6	
$f(n)$	**10,**	**15,**	**20,**	**25,**	**30,**	**35,**	**...**

MathBits.com

Compare how many 5's are added, and the term number.

Term 1: 10 ---------------------- $\rightarrow 10 + 5(0)$
Term 2: 10 + 5 ----------------- $\rightarrow 10 + 5(1)$
Term 3: 10 + 5 + 5 ------------ $\rightarrow 10 + 5(2)$
Term 4: 10 + 5 + 5 + 5 -------- $\rightarrow 10 + 5(3)$
Term 5: 10 + 5 + 5 + 5 + 5 --- $\rightarrow 10 + 5(4)$

Explicit formula: $f(n) = 10 + 5(n - 1)$

If you compare the term number with how many times the common difference, 5, is added, you will see a pattern for an explicit formula:

function notation

$$f(n) = \underbrace{f(1)}_{\text{first term}} + \overbrace{d}^{\text{common difference}} \underbrace{(n - 1)}_{\text{one less than the term number}}$$

MathBits.com

To summarize the process of writing an explicit formula for an arithmetic sequence:

1. Determine if the sequence is arithmetic (*Do you add, or subtract, the same amount from one term to the next?*)
2. Find the common difference. (*The number you add or subtract.*)
3. Create an explicit formula using the pattern of the first term added to the product of the common difference and one less than the term number.

$a_n = a_1 + d\,(n - 1)$ the "n's" shadow mean	a_n = the n^{th} term in the sequence a_1 = the first term in the sequence n = the term number d = the common difference.	$a_n = \underbrace{a_1}_{\text{first term}} + \overbrace{d}^{\text{common difference}} \underbrace{(n-1)}_{\text{one less than the term number}}$ MathBits.com
{10, 15, 20, 25, 30, 35, ...}	first term = 10, common difference = 5 explicit formula: $a_n = 10 + 5(n - 1)$ $= 10 + 5n - 5 = 5 + 5n$ or $5n + 5$	

a "sub" 1

★ Given on Regents Reference sheet

Formula to find any term of an arithmetic sequence: $y = mx + b$ $a_n = a_1 + d(n - 1)$

a_n: specific term value y; a_1: 1st term b; d: common difference m; n: specific term (x)

Examples:

1) a) Write an explicit formula in terms of n for the nth number in the following sequence.

$a_0\ a_1\ a_2\ a_3 \ldots$

0, 2, 4, 6, 8, 10, 12, 14, 16, ...

+2 +2 +2

b: y-int = a_0

$a_1 = 2$

$d = 2$

same

$a_n = a_1 + d(n-1)$

$a_n = 2 + 2(n-1)$

$a_n = 2 + 2n - 2$

$a_n = 2n$

slope +0 = y-int : b

★ Make sure to always simplify!

b) Now use the formula to find the 100th term.

$n = 100$

$a_n = 2n$

$a_{100} = 2(100)$

$\boxed{a_{100} = 200}$

For #2 & #3: Given the first term and the common difference of an arithmetic sequence, find the first four terms and the explicit formula. → must do 1st

2) $a_1 = 28$, $d = 10$

$a_n = a_1 + d(n-1)$

$a_n = 28 + 10(n-1)$

$a_n = 28 + 10n - 10$

$a_n = 10n + 18$

m: slope d; b: y-int: a_0

n = 2

$a_n = 10n + 18$

$a_2 = 10(2) + 18$

$a_2 = 20 + 18$

$\boxed{a_2 = 38}$

n = 3

$a_n = 10n + 18$

$a_3 = 10(3) + 18$

$a_3 = 30 + 18$

$\boxed{a_3 = 48}$

n = 4

$a_n = 10n + 18$

$a_4 = 10(4) + 18$

$a_4 = 40 + 18$

$\boxed{a_4 = 58}$

These #'s increase by 10 B/c d = 10

3) $a_1 = -34$, $d = -10$

$a_n = a_1 + d(n-1)$

$a_n = -34 - 10(n-1)$

$a_n = -34 - 10n + 10$

$a_n = -10n - 24$

m: slope d; b: y-int a_0

n = 2

$a_n = -10n - 24$

$a_2 = -10(2) - 24$

$a_2 = -20 - 24$

$\boxed{a_2 = -44}$

n = 3

$a_n = -10n - 24$

$a_3 = -10(3) - 24$

$a_3 = -30 - 24$

$\boxed{a_3 = -54}$

n = 4

$a_n = -10n - 24$

$a_4 = -10(4) - 24$

$a_4 = -40 - 24$

$\boxed{a_4 = -64}$

table from calc:

n	a_n
1	-34
2	-44
3	-54
4	-64

plug $y = -10x - 24$ into $\boxed{y=}$

4) a) Write an explicit formula that explains this sequence: $a_0 = 0$ 12, 24, 36, ...

$a_1 = 12$

$d = 12$

$a_n = a_1 + d(n-1)$

$a_n = 12 + 12(n-1)$

$a_n = 12 + 12n - 12$

$\boxed{a_n = 12n}$ same as $y = 12x$ ($m = 12$, $b = 0$)

$d = 12$, $a_0 = 0$

b) Now use the formula to find a_{53}

$n = 53$

$a_n = 12n$

$a_{53} = 12(53)$

$\boxed{a_{53} = 636}$

For #5 & #6: Given the explicit formula for an arithmetic sequence find the first four terms and the term named in the problem.

b: y-int, m: slope

a_0 , $d = -100$

5) $a_n = 65 - 100n$

Find a_{39}

$n=1$
$a_n = 65 - 100n$
$a_1 = 65 - 100(1)$
$a_1 = 65 - 100$
$a_1 = -35$

$n=2$
$a_n = 65 - 100n$
$a_2 = 65 - 100(2)$
$a_2 = 65 - 200$
$\boxed{a_2 = -135}$

$n=3$
$a_n = 65 - 100n$
$a_3 = 65 - 100(3)$
$a_3 = 65 - 300$
$a_3 = -235$

$n=4$
$a_n = 65 - 100n$
$a_4 = 65 - 100(4)$
$a_4 = 65 - 400$
$\boxed{a_4 = -335}$

$n=39$
$a_n = 65 - 100n$
$a_{39} = 65 - 100(39)$
$a_{39} = 65 - 3900$
$\boxed{a_{39} = -3835}$

$a_1 - a_4$: #'s decrease by 100 b/c $d = -100$

b = 0, a_0 , $d = \frac{1}{2}$, $m = \frac{1}{2}$

6) $a_n = \frac{11}{18} + \frac{1}{2}n$

Find a_{23}

$n=1$
$a_n = \frac{11}{18} + \frac{1}{2}n$
$a_1 = \frac{11}{18} + \frac{1}{2}(1)$
$a_1 = \frac{11}{18} + \frac{1}{2}$
$a_1 = 1\frac{1}{9}$

$n=2$
$a_n = \frac{11}{18} + \frac{1}{2}n$
$a_2 = \frac{11}{18} + \frac{1}{2}(2)$
$a_2 = \frac{11}{18} + 1$
$\boxed{a_2 = 1\frac{11}{18}}$

$n=3$
$a_n = \frac{11}{18} + \frac{1}{2}n$
$a_3 = \frac{11}{18} + \frac{1}{2}(3)$
$a_3 = \frac{11}{18} + \frac{3}{2}$
$\boxed{a_3 = 2\frac{1}{9}}$

$n=4$
$a_n = \frac{11}{18} + \frac{1}{2}n$
$a_4 = \frac{11}{18} + \frac{1}{2}(4)$
$a_4 = \frac{11}{18} + 2$
$\boxed{a_4 = 2\frac{11}{18}}$

$n=23$
$a_n = \frac{11}{18} + \frac{1}{2}n$
$a_n = \frac{11}{18} + \frac{1}{2}(23)$
$a_n = \frac{11}{18} + \frac{23}{2}$
$\boxed{a_n = 12\frac{1}{9}}$

7) Write the first 3 terms of the sequence whose n^{th} term is given by the explicit formula: $a_n = 2n - 1$

d, a_0
m: slope, b: y-int

$n=1$
$a_n = 2n - 1$
$a_1 = 2(1) - 1$
$a_1 = 2 - 1$
$a_1 = 1$

$n=2$
$a_n = 2n - 1$
$a_2 = 2(2) - 1$
$a_2 = 4 - 1$
$\boxed{a_3 = 3}$

$n=3$
$a_n = 2n - 1$
$a_3 = 2(3) - 1$
$a_3 = 6 - 1$
$\boxed{a_3 = 5}$

8) Find an explicit formula for the arithmetic sequence given. Then use your formula to find the indicated term: Find a_{57} when $a_{11} = 4$ and $d = 2$.

$a_{11} = 4$
$n = 11$
$d = 2$

$a_n = a_1 + d(n-1)$
$a_{11} = a_1 + 2(11-1)$
$4 = a_1 + 22 - 2$
$4 = a_1 + 20$
$-20 \quad -20$
$a_1 = -16$

$a_1 = -16$
$d = 2$
$a_n = a_1 + d(n-1)$
$a_n = -16 + 2(n-1)$
$a_n = -16 + 2n - 2$
$a_n = 2n - 18$

$m = 2$, $d = 2$; $b = -18$, $a_0 = -18$

$n = 57$
$a_n = 2n - 18$
$a_{57} = 2(57) - 18$
$a_{57} = 114 - 18$
$\boxed{a_{57} = 96}$