Name ____________________________ Date ____________

8A; Algebra 1 Period____________

Explicit Formula for Geometric Sequences

Sequence: {3, 6, 12, 24, 48, 96, ...}. Find an explicit formula.

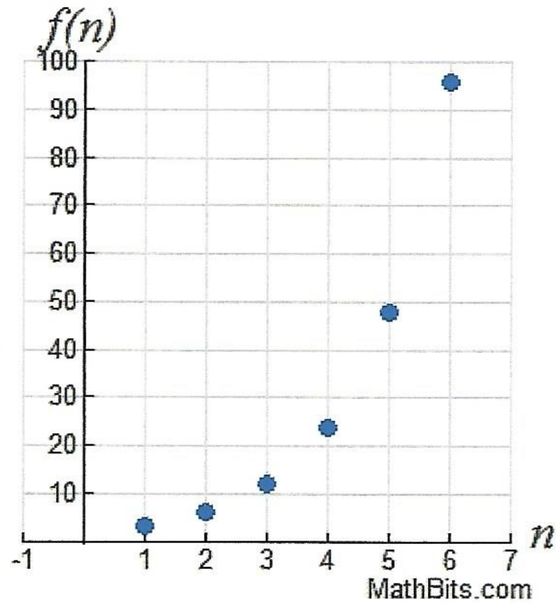
This example is a geometric sequence (the same number, 2, is multiplied times each term to get to the next term).

Term Number	Term	Subscript Notation	Function Notation
1	**3**	a_1	$f(1)$
2	**6**	a_2	$f(2)$
3	**12**	a_3	$f(3)$
4	**24**	a_4	$f(4)$
5	**48**	a_5	$f(5)$
6	**96**	a_6	$f(6)$
n	⋮	a_n	$f(n)$

Explicit Formula:

in subscript notation: $a_n = 3 \bullet (2)^{n-1}$

in function notation: $f(n) = 3 \bullet (2)^{n-1}$



Notice that this sequence has an exponential appearance. It may be the case with geometric sequences that the graph will increase (or decrease). The rate of change will increase (or decrease) as the value of n increases (it is not constant).

Will such geometric sequences be exponential functions?

Again, it is easy to see that a given explicit formula works. The problem is coming up with a formula when all you are given is a list of the terms.

If your sequence is geometric, it will help if you look at the pattern of what is happening in the sequence, in a manner similar to what we examined in the arithmetic sequence.

n	1	2	3	4	5	6	
$f(n)$	3,	6,	12,	24,	48,	96,	...

Compare powers of 2, and the term number.

Term 1: 3 ---------------- → 3×2^0

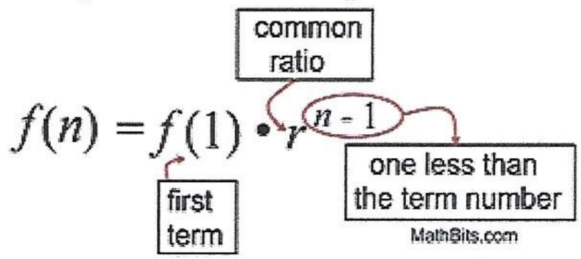
Term 2: 3×2 -------------- → 3×2^1

Term 3: 3×2×2 ----------- → 3×2^2

Term 4: 3×2×2×2 ------ → 3×2^3

Term 5: 3×2×2×2×2 --- → 3×2^4

If you compare the term number with the powers of the common difference, 2, you will see a pattern for an explicit formula:



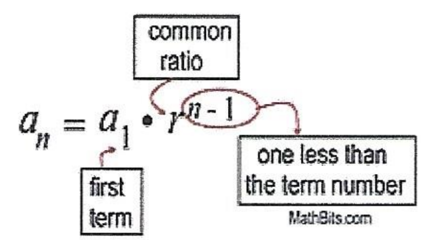
Explicit formula: $f(n) = 3 \cdot 2^{n-1}$

To summarize the process of writing an explicit formula for a geometric sequence:

1. Determine if the sequence is geometric (*Do you multiply, or divide, the same amount from one term to the next?*)

2. Find the common ratio. (*The number you multiply or divide.*)

3. Create an explicit formula using the pattern of the first term multiplied by the common ratio raised to a power of one less than the term number.

$a_n = a_1 \cdot r^{n-1}$	a_n = the n^{th} term in the sequence a_1 = the first term in the sequence n = the term number r = the common ratio	
{3, 6, 12, 24, 48, 96, ...}	first term = 10, common ratio = 2 explicit formula: $a_n = 3 \cdot 2^{n-1}$	

Formula to find any term of a geometric sequence: $a_n = a_1 \cdot r^{n-1}$

Examples:

1) Given the first term and the common ratio of a geometric sequence, find the first five terms and the explicit formula.

a) $a_1 = 4, r = 5$

b) $a_1 = 0.8$, $r = -5$

2) Write an explicit formula that explains the given sequence. Use the formula to find the indicated term.

a) 50, 10, 2, ...

Explicit formula:_____________

6th term:__________________

b) 2, -6, 18, ...

Explicit formula:_____________

12th term:_________________

3) The first term of a geometric sequence is 6 and the common ratio is -8. Find the 7th term.

4) Find the next three terms in each geometric sequence.

a) 7, 56, 448, 3584,...

b) 40, 10, $\frac{5}{2}$, $\frac{5}{8}$,...

5) What is the fifteenth term of the sequence: 5, -10, 20, -40, 80,....?

6) Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

a) $a_n = 3^{n-1}$

b) $a_n = -2.5 \bullet 4^{n-1}$

7) What is the common ratio of the geometric sequence whose first term is 27 and fourth term is 64?

a) $\frac{3}{4}$

b) $\frac{64}{81}$

c) $\frac{4}{3}$

d) $\frac{37}{3}$

8) A sequence has the following terms: $a_1 = 4$, $a_2 = 10$, $a_3 = 25$, $a_4 = 62.5$. Which formula represents the n^{th} term in the sequence?

a) $a_n = 4 + 2.5n$
b) $a_n = 4 + 2.5(n - 1)$
c) $a_n = 4(2.5)^n$
d) $a_n = 4(2.5)^{n-1}$

9) The 15th term of a geometric sequence is 32,768. Which choice shows the possible first term and the possible common ratio?

a) 2,2

b) 4,3

c) 15,4

d) 8, -4

The table shows the number of houses in a new subdivision. Use the table to answer questions 10-13.

Month	Houses
1	3
2	6
3	12
4	24

10) The number of houses forms a geometric sequence. What is r?

a) 0.5 b) 2 c) 3 d) 6

11) Assuming that the trend continues, how many houses would be in the subdivision in Month 6?

a) 36 b) 48 c) 60 d) 96

12) Management decides the subdivision is complete when the number of houses reaches 48. When will this happen?

a) Month 5 b) Month 6 c) Month 7 d) Month 8

13) Suppose the number of houses tripled every month. How many more houses would be in the subdivision in Month 4? (The number of houses in Month 1 is still 3.)

14) A student comes to school with the flu and infects three other students within an hour before going home. Each newly infected student passes the virus to three new students in the next hour. The pattern continues until all the students in the school are infected with the virus. Does this scenario describe an arithmetic or geometric sequence? Write the formula for the sequence.