Name Key
8A; Algebra 1

Date ____________
Period____________

Explicit Formula for Geometric Sequences

Sequence: {3, 6, 12, 24, 48, 96, ...}. Find an explicit formula.

This example is a geometric sequence (the same number, 2, is multiplied times each term to get to the next term).

·2 ·2 ·2 ·2

Term Number	Term Value	Subscript Notation	Function Notation
1	**3**	a_1	$f(1)$
2	**6**	a_2	$f(2)$
3	**12**	a_3	$f(3)$
4	**24**	a_4	$f(4)$
5	**48**	a_5	$f(5)$
6	**96**	a_6	$f(6)$
n	⋮	a_n	$f(n)$

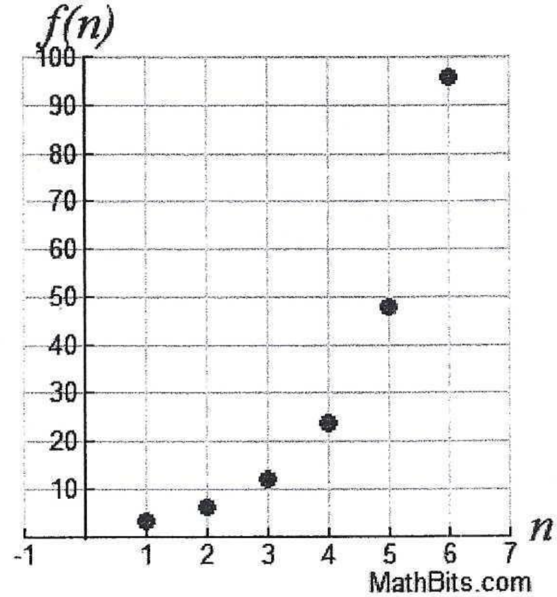
Explicit Formula:

in subscript notation: $a_n = 3 \cdot (2)^{n-1}$

in function notation: $f(n) = 3 \cdot (2)^{n-1}$

*Don't connect dots

*only Natural #'s



Notice that this sequence has an exponential appearance. It may be the case with geometric sequences that the graph will increase (or decrease). The rate of change will increase (or decrease) as the value of n increases (it is not constant).

Will such geometric sequences be exponential functions?

B/c it's related to the exponential equation Yes! $y = a(b)^x$

y-int (a_0) Base (ratio)

Again, it is easy to see that a given explicit formula works. The problem is coming up with a formula when all you are given is a list of the terms.

If your sequence is geometric, it will help if you look at the pattern of what is happening in the sequence, in a manner similar to what we examined in the arithmetic sequence.

n	1	2	3	4	5	6
$f(n)$	**3,**	**6,**	**12,**	**24,**	**48,**	**96, ...**

Compare powers of 2, and the term number.

Term 1: 3 ----------------- $\rightarrow 3 \times 2^0$

Term 2: 3×2 -------------- $\rightarrow 3 \times 2^1$

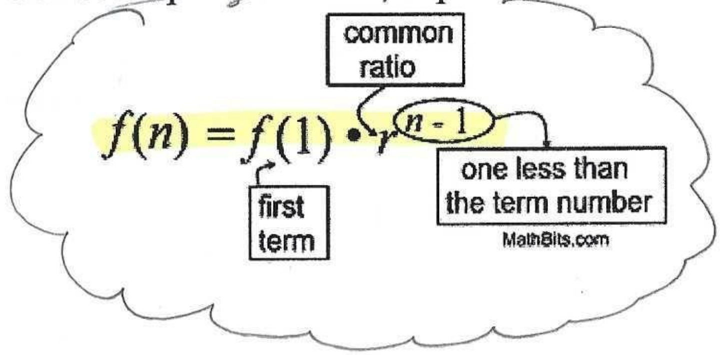
Term 3: 3×2×2 ----------- $\rightarrow 3 \times 2^2$

Term 4: 3×2×2×2 ------ $\rightarrow 3 \times 2^3$

Term 5: 3×2×2×2×2 --- $\rightarrow 3 \times 2^4$

Explicit formula: $f(n) = 3 \cdot 2^{n-1}$

If you compare the term number with the powers of the common difference, 2, you will see a pattern for an explicit formula:

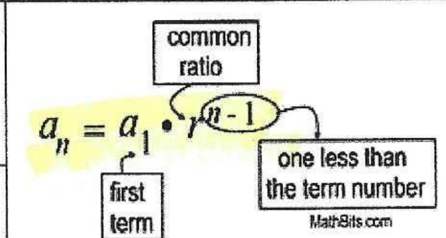


To summarize the process of writing an explicit formula for a geometric sequence:

1. Determine if the sequence is geometric (*Do you multiply, or divide, the same amount from one term to the next?*)

2. Find the common ratio. (*The number you multiply or divide.*)

3. Create an explicit formula using the pattern of the first term multiplied by the common ratio raised to a power of one less than the term number.

$a_n = a_1 \cdot r^{n-1}$	a_n = the n^{th} term in the sequence a_1 = the first term in the sequence n = the term number r = the common ratio	
{3, 6, 12, 24, 48, 96, ...}	first term = 10, common ratio = 2 explicit formula: $a_n = 3 \cdot 2^{n-1}$	

must get the formula the 1st

Formula to find any term of a geometric sequence: $a_n = a_1 \cdot r^{n-1}$

dot means multiply; specific term

specific term value; 1st term; common ratio

Examples:

1) Given the first term and the common ratio of a geometric sequence, find the first five terms and the explicit formula.

a) $a_1 = 4$, $r = 5$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 4 \cdot 5^{n-1}}$

$n=2$: $a_n = 4 \cdot 5^{n-1}$, $a_2 = 4 \cdot 5^{(2-1)}$, $a_2 = 4 \cdot 5^1$, $\boxed{a_2 = 20}$

$n=3$: $a_n = 4 \cdot 5^{n-1}$, $a_3 = 4 \cdot 5^{(3-1)}$, $a_3 = 4 \cdot 5^2$, $a_3 = 4 \cdot 25$, $\boxed{a_3 = 100}$

$n=4$: $a_n = 4 \cdot 5^{n-1}$, $a_4 = 4 \cdot 5^{(4-1)}$, $a_4 = 4 \cdot 5^3$, $a_4 = 4 \cdot 125$, $\boxed{a_4 = 500}$

$n=5$: $a_n = 4 \cdot 5^{n-1}$, $a_5 = 4 \cdot 5^{(5-1)}$, $a_5 = 4 \cdot 5^4$, $a_5 = 4 \cdot 625$, $\boxed{a_5 = 2500}$

×5 ×5 ×5

all #'s multiplied by 5 b/c r = 5

M.C. work

b) $a_1 = 0.8$, $r = -5$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = .8 \cdot (-5)^{n-1}}$

*must get the formula 1st

Must Be in ( ) B/c it's a Neg #

multiply by -5

$a_1 = .8$

$a_2 = -4$

$a_3 = 20$

$a_4 = -100$

$a_5 = 500$

$\cdot -5$

*all #'s multiplied by -5 B/c r=-5

n	a_n
1	.8
2	-4
3	20
4	-100
5	500

*get values from plugging equation in & going to the table

MC work

2) Write an explicit formula that explains the given sequence. Use the formula to find the indicated term.

a) 50, 10, 2, ...

$a_1 = 50$

$r = \frac{1}{5}$

*must do 1st

Explicit formula: $a_n = 50 \cdot (\frac{1}{5})^{n-1}$

$n=6$

6th term: $\frac{2}{125}$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 50 \cdot (\frac{1}{5})^{n-1}$

$n=6$

$a_n = 50 \cdot (\frac{1}{5})^{(n-1)}$

$a_6 = 50 \cdot (\frac{1}{5})^{(6-1)}$

$a_6 = 50 \cdot (\frac{1}{5})^{5}$

$a_6 = 50 \cdot \frac{1}{3125}$

$a_6 = \frac{2}{125}$

b) 2, -6, 18, ...

$a_1 = 2$

$r = -3$

*must be in ( ) B/c it is a Neg #!

*must do 1st

Explicit formula: $a_n = 2 \cdot (-3)^{n-1}$

$n=12$

12th term: $-354,294$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 2 \cdot (-3)^{n-1}$

must have ( )

$n=12$

$a_n = 2 \cdot (-3)^{n-1}$

$a_{12} = 2 \cdot (-3)^{(12-1)}$

$a_{12} = 2 \cdot (-3)^{11}$

*must be in ( ) when putting into calc

$a_{12} = 2 \cdot -177,147$

$a_{12} = -354,294$

3) The first term of a geometric sequence is 6 and the common ratio is -8. Find the 7th term.

a_1 $r=-8$ $n=7$

*must find the 1st formula

$a_1 = 6$

$r = -8$

$n = 7$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 6 \cdot (-8)^{n-1}}$

$a_n = 6 \cdot (-8)^{n-1}$

$a_7 = 6 \cdot (-8)^{(7-1)}$

$a_7 = 6 \cdot (-8)^6$

$a_7 = 6 \cdot 262144$

$\boxed{a_7 = 1,572,864}$

4) Find the next three terms in each geometric sequence.

*must get formula 1st

a) 7, 56, 448, 3584,... a_5 ___ a_6 ___ a_7 ___

$a_1 = 7$

$r = 8$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 7 \cdot (8)^{n-1}}$

$n=5$

$a_n = 7 \cdot (8)^{n-1}$

$a_5 = 7 \cdot (8)^{(5-1)}$

$a_5 = 7 \cdot (8)^4$

$a_5 = 7 \cdot 4096$

$\boxed{a_5 = 28,672}$

$n=6$

$a_n = 7 \cdot (8)^{n-1}$

$a_6 = 7 \cdot (8)^{(6-1)}$

$a_6 = 7 \cdot 8^5$

$a_6 = 7 \cdot 32768$

$\boxed{a_6 = 229,376}$

$n=7$

$a_n = 7 \cdot 8^{n-1}$

$a_7 = 7 \cdot 8^{(7-1)}$

$a_7 = 7 \cdot 8^6$

$a_7 = 7 \cdot 262144$

$\boxed{a_7 = 1,835,008}$

M.C. *must get formula 1st

b) 40, 10, $\frac{5}{2}$, $\frac{5}{8}$,... $\frac{5}{32}$, $\frac{5}{128}$, $\frac{5}{512}$

$a_1 = 40$

$r = \frac{1}{4}$

$\boxed{a_n = 40 \cdot \frac{1}{4}^{n-1}}$

M.C. work

n	a_n
5	.15625
6	.03906
7	.00977

5) What is the fifteenth term of the sequence: 5, -10, 20, -40, 80,....?

must get the formula 1st

$n = 15$
$a_1 = 5$
$r = -2$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 5 \cdot (-2)^{n-1}}$

must be in () b/c it's neg

$a_n = 5 \cdot (-2)^{n-1}$
$a_{15} = 5 \cdot (-2)^{(15-1)}$ must put in () in calc or it won't register the neg
$a_{15} = 5 \cdot (-2)^{14}$
$a_{15} = 5 \cdot 16384$
$\boxed{a_{15} = 81,920}$

6) Given the explicit formula for a geometric sequence find the first five terms and the 8th term.

a) $a_n = 3^{n-1}$

$a_n = 1 \cdot 3^{n-1}$ same $r = 3$

$n = 1$: $a_n = 3^{n-1}$, $a_1 = 3^{(1-1)}$, $a_1 = 3^0$, $\boxed{a_1 = 1}$

$n = 2$: $a_n = 3^{n-1}$, $a_2 = 3^{(2-1)}$, $a_2 = 3^1$, $a_2 = 3$

$n = 3$: $a_n = 3^{n-1}$, $a_3 = 3^{(3-1)}$, $a_3 = 3^2$, $a_3 = 9$

$n = 4$: $a_n = 3^{n-1}$, $a_4 = 3^{(4-1)}$, $a_4 = 3^3$, $\boxed{a_4 = 27}$

$n = 5$: $a_n = 3^{n-1}$, $a_5 = 3^{(5-1)}$, $a_5 = 3^4$, $\boxed{a_5 = 81}$

$n = 8$: $a_n = 3^{n-1}$, $a_8 = 3^{(8-1)}$, $a_8 = 3^7$, $a_8 = 2187$

b) $a_n = -2.5 \cdot 4^{n-1}$

a_1, r

$a_1 = -2.5$
$r = 4$

M.C. work

$a_1 = -2.5$ $a_4 = -160$
$a_2 = -10$ $a_5 = -640$
$a_3 = -40$ $a_8 = -40,960$

get values from plugging equation into $y=$ & going to table

n	a_n
1	-2.5
2	-10
3	-40
4	-160
5	-640
8	-40,960

7) What is the common ratio of the geometric sequence whose first term is 27 and fourth term is 64?

$a_1 = 27$ $a_4 = 64$

r

a) $\cancel{\frac{3}{4}}$ b) $\cancel{\frac{64}{81}}$ c) $\frac{4}{3}$ d) $\frac{37}{3}$

guess + check

27, 36, 48, 64

#'s get Bigger = multiply by # Bigger than 1

$a_1 = 27$
$a_4 = 64$
$r = ?$

$n = 4$
$a_n = a_1 \cdot r^{n-1}$
$a_4 = 27 \cdot r^{4-1}$
$64 = 27 \cdot r^{4-1}$
$\frac{64}{27} = \frac{27 \cdot r^3}{27}$
$\sqrt[3]{r^3}$ $\frac{\sqrt[3]{64}}{\sqrt[3]{27}}$
$\boxed{r = \frac{4}{3}}$

8) A sequence has the following terms: $a_1 = 4$, $a_2 = 10$, $a_3 = 25$, $a_4 = 62.5$. Which formula represents the n^{th} term in the sequence?

a) ~~$a_n = 4 + 2.5n$~~
b) ~~$a_n = 4 + 2.5(n - 1)$~~
c) $a_n = 4(2.5)^n$
d) $a_n = 4(2.5)^{n-1}$

geometric

$a_1 = 4$
$r = 2.5$

$a_n = a_1 \cdot r^{n-1}$
$a_n = 4 \cdot (2.5)^{n-1}$

9) The 15th term of a geometric sequence is 32, 768. Which choice shows the possible first term and the possible common ratio?

$a_{15} = 32,768$

a) 2,2 b) 4,3 c) 15,4 d) ~~8, -4~~

$a_1 = 2$
$r = 2$
$a_n = a_1 \cdot r^{n-1}$
$a_n = 2 \cdot 2^{n-1}$
$a_n = 2^{1+n-1}$
$\boxed{a_n = 2^n}$

can add exponents of like bases

$n = 15$
$a_n = 2^n$
$a_{15} = 2^{15}$
$a_{15} = 32,768$

✓ checks out

keep the base the same
Add exponents of like bases

13

The table shows the number of houses in a new subdivision. Use the table to answer questions 10-~~12~~13.

$a_1 \rightarrow$

Month	Houses
1	3
2	6
3	12
4	24
5	48
6	96

$\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$

double $r=2$

$r=2$

10) The number of houses forms a geometric sequence. What is r?

a) 0.5 b) 2 c) 3 d) 6

11) Assuming that the trend continues, how many houses would be in the subdivision in Month 6?

a) 36 b) 48 c) 60 d) 96

$a_1 = 3$, $r = 2$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 3 \cdot 2^{n-1}$

$n=6$

$a_n = 3 \cdot 2^{n-1}$ $n=6$

$a_6 = 3 \cdot 2^{(6-1)}$

$a_6 = 3 \cdot 2^5$

$a_6 = 3 \cdot 32$

$a_6 = 96$

) Management decides the subdivision is complete when the number of houses reaches 48. When will this happen?

a) Month 5 b) Month 6 c) Month 7 d) Month 8

$n=5$

guess + check

$a_n = 3 \cdot 2^{n-1}$

$a_5 = 3 \cdot 2^{(5-1)}$

$a_5 = 3 \cdot 2^4$

$a_5 = 3 \cdot 16$

$a_5 = 48$

$r=3$ subtract

Add exponents of like Bases

13) Suppose the number of houses tripled every month. How many more houses would be in the subdivision in Month 4? (The number of houses in Month 1 is still 3.)

Month	Houses
1	3
2	9
3	27
4	81

$r=3$

$1 = a_0$

$81 - 24 = \boxed{57}$ more

$a_1 = 3$, $r = 3$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 3 \cdot 3^{n-1}$

$a_n = 3^{1+n-1}$

$a_n = 3^n$

$a_n = 3^n$ $n=4$

$a_4 = 3^4$

$a_4 = 81$

$a_1 = 3$

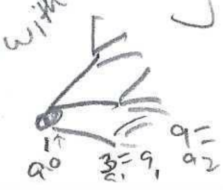
14) A student comes to school with the flu and infects three other students within an hour before going home. Each newly infected student passes the virus to three new students in the next hour. The pattern continues until all the students in the school are infected with the virus. Does this scenario describe an arithmetic or geometric sequence? Write the formula for the sequence.

start with a beginning $= a_0$

geometric

$r=3$

$a_1 = 3 \rightarrow$ # of kids he infects



$a_n = a_1 \cdot r^{n-1}$

$a_n = 3 \cdot 3^{n-1}$

$a_n = 3^{1+n-1}$

$\boxed{a_n = 3^n}$

keep base + add exponents of like bases

Simplify

(1) $5 \cdot 5^{x-2}$

5^{1+x-2}

$\boxed{5^{x-1}}$

★ keep base!

+ add exponents

(2) $4 \cdot \underline{2}^{n-2}$

$2^{2} \cdot 2^{n-2}$

2^{2+n-2}

$\boxed{2^{n}}$

★ must have like Bases, so make them like bases

(3) $27 \cdot 3^{n+3}$

$3^{3} \cdot 3^{n+3}$

3^{3+n+3}

$\boxed{3^{n+6}}$

must have like Bases, so make them like bases