

## Measures of Central Tendency

### I. Definitions:

**Mean:** Arithmetic average. You add up all of the values and divide by the number of values.

**Median:** The middle value, or the average of the two middle numbers when the numbers are arranged in numerical order.

**Mode:** The data value that appears most often in the set. You can have more than one mode or no mode at all.

**Range:** The <sup>Subtract</sup> difference between the highest and the lowest values in a set of data.

### How do I know which measure of central tendency to use?

#### MEAN

Use the mean to describe the middle of a set of data that does not have an outlier.

#### Advantages:

- Most popular measure in fields such as business, engineering and computer science.
- It is unique - there is only one answer.
- Useful when comparing sets of data.

#### Disadvantages:

- Affected by extreme values (outliers)

#### MEDIAN

Use the median to describe the middle of a set of data that does have an outlier.

#### Advantages:

- Extreme values (outliers) do not affect the median as strongly as they do the mean.
- Useful when comparing sets of data.
- It is unique - there is only one answer.

#### Disadvantages:

- Not as popular as mean.

#### MODE

Use the mode when the data is non-numeric or when asked to choose the most popular item.

#### Advantages:

- Extreme values (outliers) do not affect the mode.

#### Disadvantages:

- Not as popular as mean and median.
- Not necessarily unique - may be more than one answer
- When no values repeat in the data set, the mode is every value and is useless.
- When there is more than one mode, it is difficult to interpret and/or compare.

What will happen to the measures of central tendency if we add the same amount to all data values, or multiply each data value by the same amount?

	Data	Mean	Mode	Median
Original Data Set:	6, 7, 8, 10, 12, 14, 14, 15, 16, 20	12.2	14	13
Add 3 to each data value	9, 10, 11, 13, 15, 17, 17, 18, 19, 23	15.2	17	16
Multiply 2 times each data value	12, 14, 16, 20, 24, 28, 28, 30, 32, 40	24.4	28	26

**When added:** Since all values are shifted the same amount, the measures of central tendency all shifted by the same amount. If you add 3 to each data value, you will add 3 to the mean, mode and median.

**When multiplied:** Since all values are affected by the same multiplicative values, the measures of central tendency will feel the same affect. If you multiply each data value by 2, you will multiply the mean, mode and median by 2.

## II. Example:

- 1) Ms. Mosher recorded the math test scores of six students in the table below.

Student	Student Score
Andrew	72
John	80
George	85
Amber	93
Betty	78
Roberto	80

- (a) Determine the **mean** of the student scores, to the **nearest tenth**. [Show all work.]
- (b) Determine the **median** of the student scores. [Show all work.]
- (c) Describe the effect on the mean and the median if Ms. Mosher adds 5 bonus points to each of the six students scores.

Mean on the calc

Calc: (STAT) (1: Edit) (P) (1: Intv) (L) (press enter after each #)

(STAT) (=>) (CALC) (1: 1-var stats) (Calculate) =

$\bar{x} = \text{Mean}$

$\sum x = \text{Sum}$   
 $n = \text{# of data values}$

Median: same steps as above + then scroll down range to!

$\sum x = \text{sum on the calc}$   
 $\frac{488}{n=6} = 81.\bar{3}$  81.3

72, 78, 80, 80, 85, 93  $\frac{160}{2} = 80$

Mean: The mean will increase by 5 + will now be 86.3

Median: The median will increase by 5 + will now be 85

Practice Examples:

1) Find the mean, median, and mode for the following data.

5, 15, 10, 15, 5, 10, 10, 20, 25, 15

*and range*

*max-min  
25-5*

$\frac{\sum x}{n} = \frac{130}{10}$   
 $\bar{x}$  mean = 13    median = 12.5    mode = 10 + 15    range = 20

\*Which measure of central tendency best represents the data above? mean (B/c of no outliers)

2) Find the median for each distribution.

(a) 3, 2, 5, 5, 1

(b) 9, 8, 8, 7, 4, 3, 3, 2, 0, 0

median = 3

median = 3.5

3) Find the mode for each distribution

(a) 2, 9, 3, 7, 3

(b) 3, 4, 5, 4, 3, 7, 2

(c) 1, 2, 3, 4, 5, 6, 7

mode = 3

mode = 3 + 4

mode = no mode

4) Alyssa received the mark of  $x$  on two of her tests and  $y + 7$  on each of three other tests. Represent her average for all five tests in terms of  $x$  and  $y$ .

$$\frac{x + x + y + 7 + y + 7 + y + 7}{5}$$

$$\frac{2x + 3y + 21}{5}$$

5) The mean weight of three dogs is 38 pounds. One of the dogs, Roxy, weighs 46 pounds. The other two dogs, Zoey, and Lexi, have the same weight. Find Zoey's weight.

let  $x =$  Zoey's + Lexi's weight

$$\frac{46 + x + x}{3} = 38$$

$$46 + 2x = 38 \cdot 3$$

$$46 + 2x = 114$$

$$2x = 68$$

$$x = 34$$

$$\frac{46 + 2x}{3} = 38$$

$$46 + 2x = 114$$

$$2x = 68$$

$$x = 34$$

Zoey's weight is 34 lbs.

$$\frac{46 + 34 + 34}{3} = 38$$

6) For what value of  $x$  will 8 and  $x$  have the same mean as 27 and 5?

$$\frac{8+x}{2} = \frac{27+5}{2}$$

$$\frac{8+x}{2} = \frac{32}{2}$$

$$8+x = 32$$

$$x = 24$$

$$2(8+x) = 64$$

$$16+2x = 64$$

$$-16 \quad -16$$

$$\frac{2x}{2} = \frac{48}{2}$$

$$x = 24$$

7) On his first 5 biology tests, Bob received the following scores: 72, 86, 92, 63, and 77. What test score must Bob earn on his sixth test so that his average (mean score) for all six tests will be 80? You must show work.

Let  $x =$  Bob's 6<sup>th</sup> test score

$$\frac{72+86+92+63+77+x}{6} = 80$$

$$390+x = 480$$

$$-390 \quad -390$$

$$x = 90$$

The 6<sup>th</sup> score must be a 90.

$$\frac{72+86+92+63+77+x}{6} = 80$$

$$\frac{490}{6} = 80$$

$$80 = 80$$

8) The average of weights of John, Chris, and Nick is 75 kilograms. How much does Daniel weigh if the mean weight of the four boys is 80 kilograms?

Let  $x =$  Daniel's weight

$$\frac{75+75+75+x}{4} = 80$$

$$225+x = 320$$

$$-225 \quad -225$$

$$x = 95$$

Dan's weight is 95 kg.

$$\frac{75+75+75+95}{4} = 80$$

$$\frac{320}{4} = 80$$

$$80 = 80$$

9) The average of three consecutive even integers is 20. Find the integers.

Let  $x =$  1<sup>st</sup> CEI  
 $x+2 =$  2<sup>nd</sup> CEI  
 $x+4 =$  3<sup>rd</sup> CEI

$$\frac{x+x+2+x+4}{3} = 20$$

$$\frac{3x+6}{3} = \frac{20}{1}$$

$$3x+6 = 60$$

$$-6 \quad -6$$

$$\frac{3x}{3} = \frac{54}{3}$$

$$x = 18$$

$$x+2 = 20$$

$$x+4 = 22$$

The three CEI's whose avg. is 20 are 18, 20, 22

$$\frac{18+20+22}{3} = 20$$

$$\frac{60}{3} = 20$$

$$20 = 20$$

10) The average of four numbers is 42. The second number is twice the first number, the third number is three less than the second, and the fourth is three more than three times the first. Find the four numbers.

Let  $x =$  1<sup>st</sup> #  
 $2x =$  2<sup>nd</sup> #  
 $2x-3 =$  3<sup>rd</sup> #  
 $3x+3 =$  4<sup>th</sup> #

$$\frac{x+2x+2x-3+3x+3}{4} = 42$$

$$\frac{8x}{4} = \frac{42}{1}$$

$$\frac{8x}{8} = \frac{168}{8}$$

$$x = 21$$

$$2x = 42$$

$$2x-3 = 39$$

$$3x+3 = 66$$

The four #'s whose avg is 42 are 21, 42, 39, 66

$$\frac{21+42+39+66}{4} = 42$$

$$\frac{168}{4} = 42$$

$$42 = 42$$