Name: Key Date:

Solving Linear Equations with Variables on Both Sides
Algebra 1

Often we will need to solve linear equations where the variable happens to be on both sides of the equality. The objective in solving these equations is the same as that of the simpler ones that we have seen – isolate the variable and solve for its value. The key is, as always, to manipulate the equation by doing the same thing to both of its sides. → Equality properties

goal

***Exercise* #1:** Solve the linear equation below and check your answer. List the properties used.

★ Start by moving the smaller variable to the larger (so you don't have neg #'s)
★ Use opposite operation b/c it's on the opposite side

$5x-5=2x+13$
$-2x \quad -2x$
$3x-5=13$
$+5 \quad +5$
$\frac{3x}{3}=\frac{18}{3}$
$x=6$

Check (PEMDAS)
$5x-5=2x+13$
$5(6)-5=2(6)+13$
$30-5=12+13$
$25=25$ ✓

As we see from this exercise, an equation truly works like a balancing scale. As long as we perform the same operation to both sides of the equation, like adding the same amount of weight to both sides of a scale, the equation remains valid - the scale remains balanced.

***Exercise* #2:** Solve each of the following equations. Check and list the properties used.

(a) $7x-21=x+3$
$-1x \quad -1x$ Sub Prop of =
$6x-21=3$ Add Prop of =
$+21 \quad +21$
$\frac{6x}{6}=\frac{24}{6}$ Div Prop of =
$x=4$

(b) $-4+3x=6x+32$
$-3x \quad -3x$ Sub Prop of =
$-4=3x+32$ Sub Prop of =
$-32 \quad -32$
$\frac{-36}{3}=\frac{3x}{3}$ D.P.O.E
$-12=x$ $x=-12$

(c) $-2x-18=-4x-6$
$+4x \quad +4x$ Add Prop of =
$2x-18=-6$ Add Prop of =
$+18 \quad +18$
$\frac{2x}{2}=\frac{12}{2}$ Div Prop of Equal.
$x=6$

Sometimes we encounter problems where we need to combine like terms as well. It is advisable to combine first.

C = Combine like terms (same side, same operation)
M = Move smaller variable to larger (opp. side opp. operation)
S = Solve remaining equation

***Exercise* #3:** Solve each of the following equations. Check and list the properties used.

(a) $10-7x+x=5x-80-2x$
$10-6x=3x-80$ Combine like terms
$+6x \quad +6x$ Add. Prop of =
$10=9x-80$
$+80 \quad +80$ Add. Prop of =
$\frac{90}{9}=\frac{9x}{9}$ Div. Prop of =
$10=x$ $x=10$

(b) $10x-3-8x=-x+11-3x+10$
$2x-3=-4x+21$ combine like terms
$+4x \quad +4x$ A.P.O.E.
$6x-3=21$
$+3 \quad +3$ A.P.O.E.
$\frac{6x}{6}=\frac{24}{6}$ D.P.O.E.
$x=4$

Exercise #4: Which of the following values of x satisfies the equation $2x-14=7x+6$?

(1) $x=-2$ (3) $x=5$

(2) $x=6$ (4) $x=-4$

Do on calc 1st to show

$-2x \quad -2x$

$-14 = 5x + 6$

$-6 \quad -6$

$-20 = 5x$

$\frac{-20}{5} = \frac{5x}{5}$ $x=-4$

It is important to continue our work with translating verbal phrases into equations. Some of these can also result in variables on both sides of the equation.

Exercise #5: Translate each of the following verbal sentences into an equation and then solve for the number described.

(a) Eight times a number is 36 more than twice the same number.

let x = the #

$8x = 2x + 36$

$-2x \quad -2x$

$6x = 36$

$\frac{6x}{6} = \frac{36}{6}$ $x=6$

the # is 6.

(b) If three times a number is increased by 24, the result is six times the same number.

let x = the #

$3x + 24 = 6x$

$-3x \quad -3x$

$24 = 3x$

$\frac{24}{3} = \frac{3x}{3}$ $x=8$

The # is 8.

(c) If three times a number is increased by 22, the result is 14 less than seven times the same number.

let x = the #

$3x + 22 = 7x - 14$

$-3x \quad -3x$

$22 = 4x - 14$

$+14 \quad +14$

$36 = 4x$

$x = 9$

the # is 9

Exercise #6: A square and a rectangle are shown below with side lengths in terms of x. The perimeter of the rectangle is 123 more than the perimeter of the square. Find the value of x.

ADD all the sides

$3x+3x+2x+2x = x+x+x+x+123$

$10x = 4x + 123$

$-4x \quad -4x$

$6x = 123$

$\frac{6x}{6} = \frac{123}{6}$

$x = 20\frac{1}{2}$

* If the rectangle is heavier add the # to the square instead so they will be equal