*Do desmos!!

Name Key

Mrs. Roumbos

Date____________

8A Period_______

Graphing exponential functions

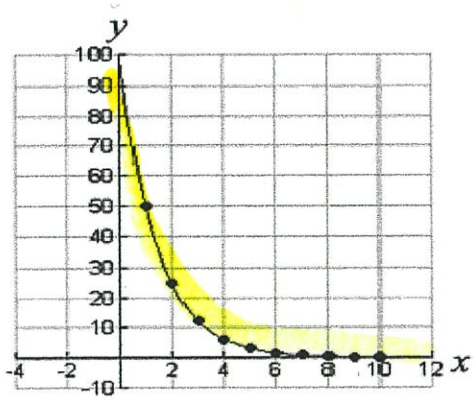
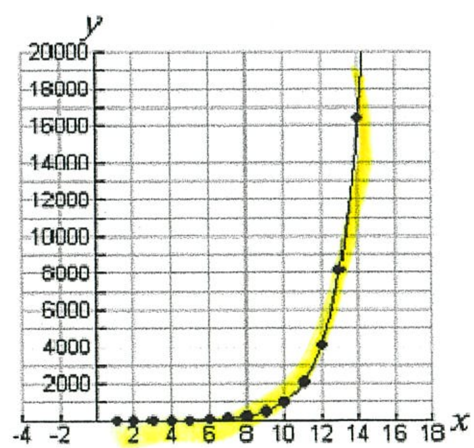
An exponential function: $y = ab^x$ ($a=1$). The basic graph of this function is a curve that approaches the negative x-axis , but never crosses it. (Only in quadrants I & II) The graph always crosses the y-axis at (0,1). Changing the coefficient of x (the exponent) to a negative number reverses the direction of the curve, where negating the base would reflect the graph (turn upside down) into quadrant III & IV. The base of the exponent will determine if the graph has growth or decay (see below)

parent function

ex $y = 2^{-x}$ Reflection over y: growth ↔ decay

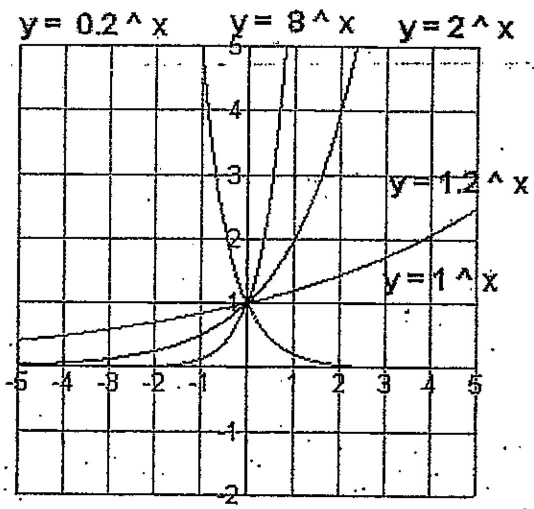
$y = -2^x$ (Reflection over x-axis)

Observe how the graphs of exponential functions change based upon the values of *a* and *b*:

$y = a \cdot b^x$	$y = a \cdot b^x$
Example: $y = 100 \cdot (0.5)^x$ y-intercept	Example: $y = 1 \cdot (2)^x$ y-intercept
when $a > 0$ and the b is between 0 and 1, the graph will be decreasing (decaying).	when $a > 0$ and the b is greater than 1, the graph will be increasing (growing).
For this example, each time x is increased by 1, y decreases to one half of its previous value.	For this example, each time x is increased by 1, y increases by a factor of 2.
	
Such a situation is called **Exponential Decay**.	Such a situation is called **Exponential Growth.**

These graphs are **ALWAYS in quadrants I & II.

Exponential Functions:



Notice that all of the exponential graphs pass through the point (0,1). This occurs because values raised to the zero power equal 1.

Exponential functions have the variable x as an exponent.
Form being examined is $y = a^x$.

Some Observations:

1. As the base value (a) gets larger, the graph becomes steeper faster - it appears to stretch upward more quickly.

2. As the base value (a) gets closer to 1, the graph flattens. If the base were to become one, the graph would be a horizontal (flat) straight line (not an exponential graph).

3. If the base value (a) is between 0 and 1, the graph appears to have reflected itself over the y-axis. The graph has just turned from exponential growth to exponential decay.

VOCABULARY

An **exponential function** is of the form $f(x) = ab^x$, where $a \neq 0$ and $b > 0$, $b \neq 1$.

Exponential growth involves the exponential increase of a quantity over time, which is represented by $y = a \cdot b^x$, where $a > 0$ and $b > 1$.

Exponential decay involves the exponential decrease of a quantity over time, which is represented by $y = a \cdot b^x$, where $a > 0$ and $0 < b < 1$.

Exponential decay often involves finding the **half-life** of an object, which is the time needed for an amount of a substance to decrease by one-half.

Example 1

$y = 2^x$ (→ Growth) *or* $f(x) = 2^x$. Graph approaches but does not cross the negative x-axis. The y-intercept is (0, 1)

Only in Quad I + II

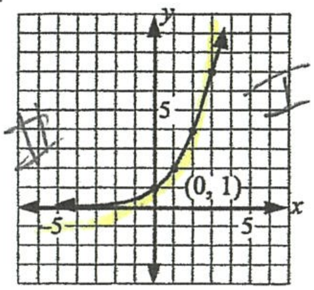


Table of Values $f(x) = 2^x$

x	y
−3	0.125
−2	0.25
−1	0.5
0	1
1	2
2	4
3	8

→ y-int

x2 → These #'s double b/c of the base of 2

Example 2

→ Reflection in the y-axis (change direction from growth to decay + vice-versa)

$y = 2^{(-x)}$ In this example, the coefficient of x is −1. The graph decreases as it goes from left to right. It has (0, 1) as its y-intercept, and now it approaches but never crosses the positive x-axis.

B/C $2^{-x} \Rightarrow \frac{1}{2^x} = .5^x$ Decay

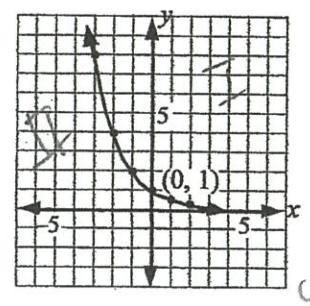


Table of Values for $y = 2^{(-x)}$

x	y
−3	8
−2	4
−1	2
0	1
1	0.5
2	0.25
3	0.125

y-int

$y = -2^x$ Reflection over the x-axis

Part I Ex's

1) Given $y = 3^x$, evaluate y when $x = 3$

$y = 3^x$

$y = 3^3$

$\boxed{y = 27}$

2) Would the graph of $y = 0.5^x$ (parent function) show exponential growth or exponential decay?

Decay B/c the base (b) is between 0 + 1

3) Which ordered pair represents the y-intercept for the function $y = 2^x$ (parent function)?

(0, 1)

B/c it's really $y = 1 \cdot 2^x$ (y-int)

4) The graph of $y = 2^x$ (parent function) lies in which Quadrants?

I + II

Graph the following:

1) Draw the graph of the equation $y = 2^x$ in the interval [-3,3]

growth

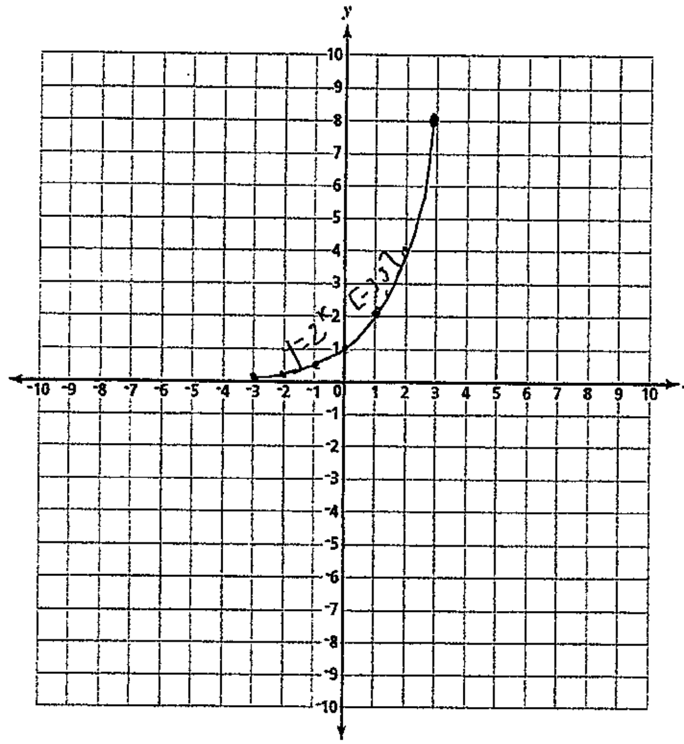
parent function

This constraint is in interval notation

* No arrows b/c of constraints given

X	Y
-3	.125
-2	.25
-1	.5
0	1
1	2
2	4
3	8

y-values are doubling b/c of the base of 2



2) Draw the graph of the equation $y = \left(\frac{1}{2}\right)^x$ in the interval [-3,5] → Constraint in interval notation

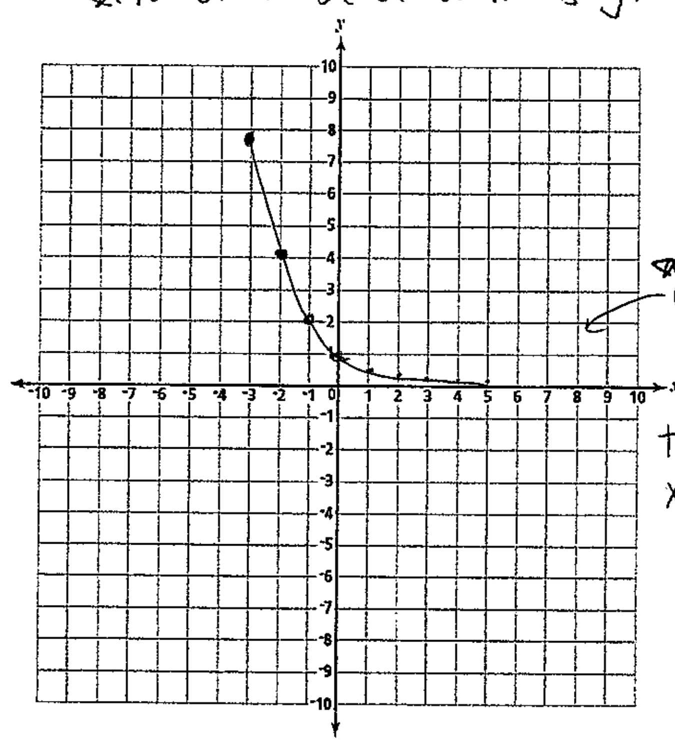
decay

* no arrows b/c of constraints given

X	Y
-3	8
-2	4
-1	2
0	1
1	.5
2	.25
3	.125
4	.0625
5	.03125

* only in Quad I + II

y-values are halved b/c of the base of $\frac{1}{2}$



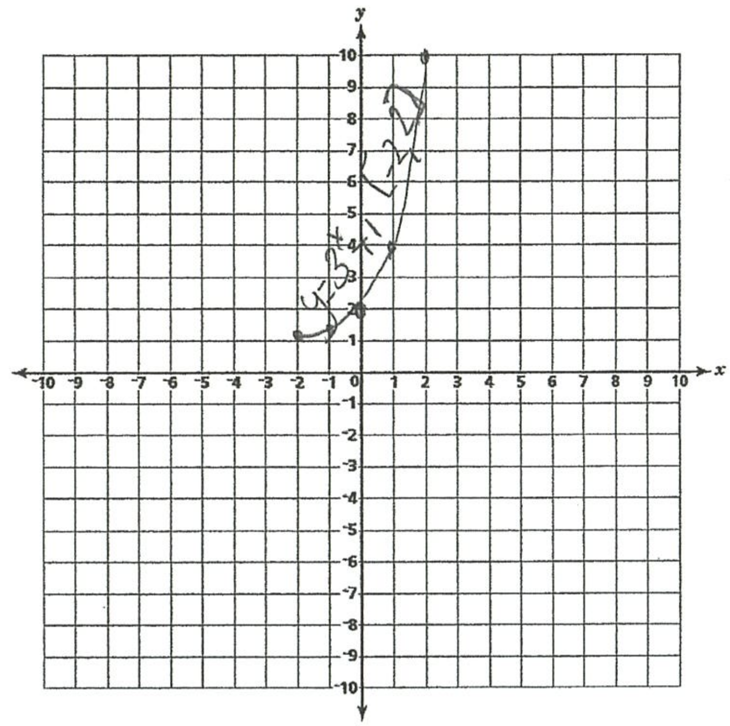
** curve can't touch the x-axis

3) $y = 3^x + 1$ ← "outside" $[-2,2]$ → interval notation → constrains = ★ no arrows

↓ growth

X	Y
-2	$1.\overline{1}$
-1	$1.\overline{3}$
0	2
1	4
2	10

→ y-int



Explain the transformation: Translated 1 unit up from (0,1) → (0,2)

4) $y = 2^{x-3}$ ← "inside"

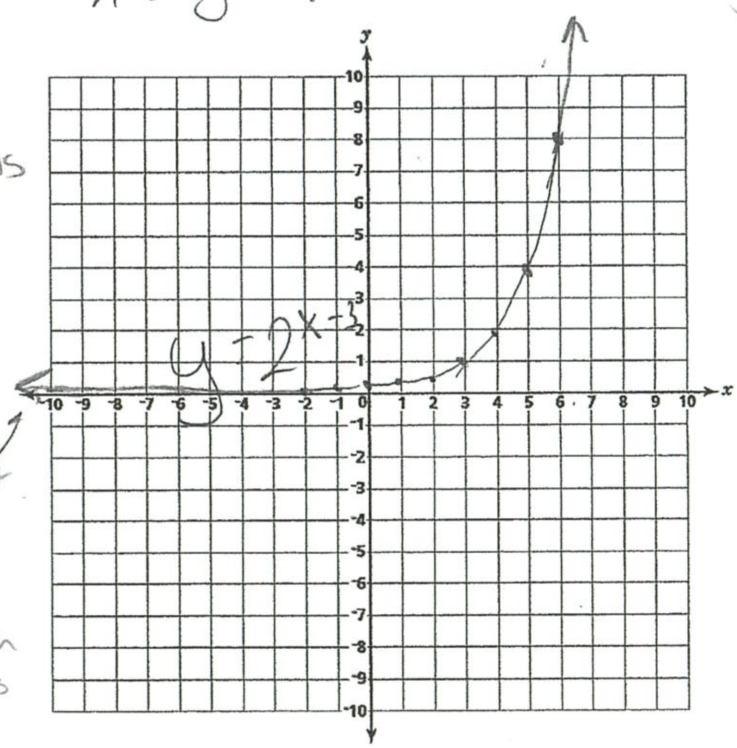
↓ growth

X	Y
-2	.03125
-1	.0625
0	.125
1	.25
2	.5
3	1
4	2
5	4
6	8

→ y-int

★ arrows B/C no constraints given

★ only in Quad I + II



★ Don't go into Quad 3 and can't touch the x-axis

Explain the transformation: Translated 3 units Right from (0,1)

(0,1) → (3,1)

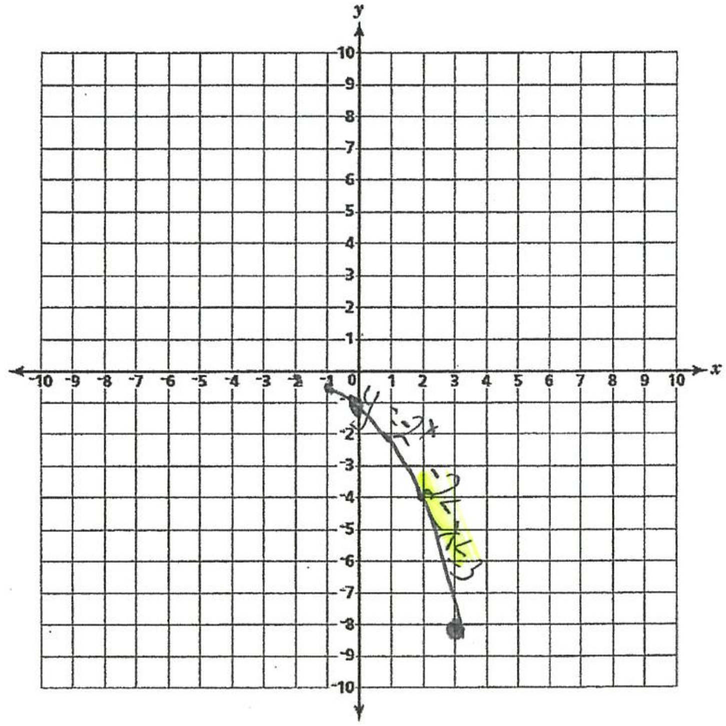
5) $y = -2^x$ $\quad -2 \le x \le 3$

outside

★ set builder ★
★ no arrows
B/c constraints given

X	Y
-2	-.25
-1	-.5
0	-1
1	-2
2	-4
3	-8

→ y-int



Explain the transformation: Reflection in the x-axis $(0,1) \to (0,-1)$

Growth "inside" Decay

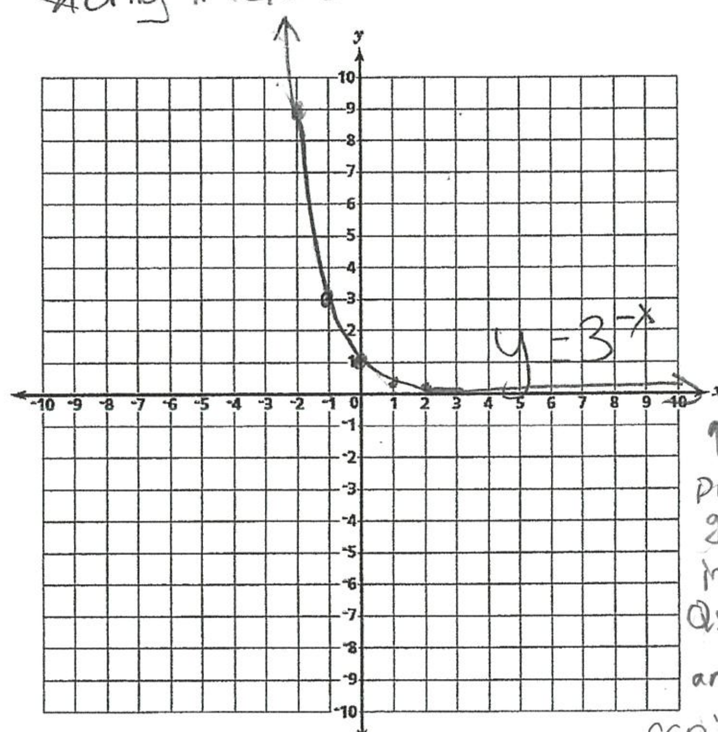
6) $y = 3^{-x} \to \frac{1}{3^x} = \frac{1}{3^x}$

X	Y
-2	9
-1	3
0	1
1	$.\overline{3}$
2	$.\overline{1}$
3	.03704

→ y-int

★ arrows B/c no constraints

★ only in Quad I + II



Don't go into Quad IV and it can't touch the x-axis

Explain the transformation: Reflection in the y-axis

★ so it changes from growth to decay