MathBitsNotebook Algebra 1

Transformations of Functions

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If you need to review your transformation skills, see Symmetry, Reflections, Translations, Dilations and Rotations.

The transformations you have seen in the past can also be used to move and resize graphs of functions. We will be examining the following changes to $f(x)$:

$-f(x)$, $f(-x)$, $f(x)+k$, $f(x+k)$, $kf(x)$, $f(kx)$

reflections translations dilations

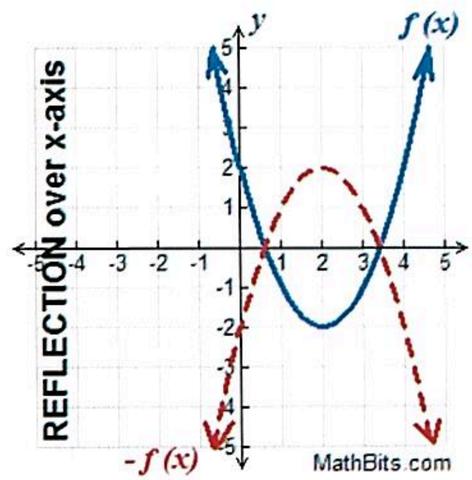
Reflections of Functions: $-f(x)$ and $f(-x)$

- Reflection over the x-axis.

$-f(x)$ reflects $f(x)$ over the x-axis

Reflections are mirror images. Think of "folding" the graph over the x-axis.

On a grid, you used the formula $(x,y) \rightarrow (x,-y)$ for a reflection in the x-axis, where the y-values were negated. Keeping in mind that $y = f(x)$, we can write this formula as $(x, f(x)) \rightarrow (x, -f(x))$.

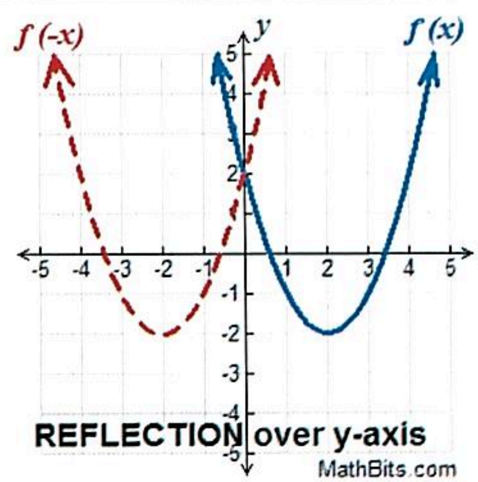


- Reflection over the y-axis.

$f(-x)$ reflects $f(x)$ over the y-axis

Reflections are mirror images. Think of "folding" the graph over the y-axis.

On a grid, you used the formula $(x,y) \rightarrow (-x,y)$ for a reflection in the y-axis, where the x-values were negated. Keeping in mind that $y = f(x)$, we can write this formula as $(x, f(x)) \rightarrow (-x, f(-x))$.



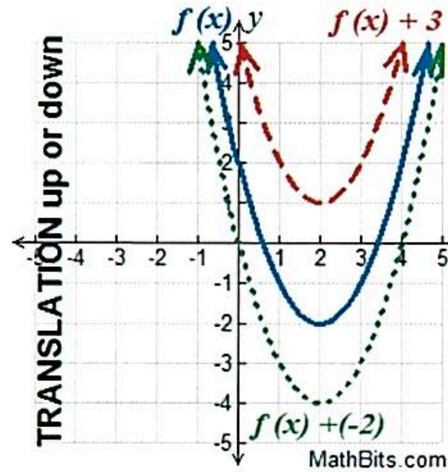
Translations of Functions: $f(x) + k$ and $f(x + k)$

● Translation vertically (upward or downward)

$f(x) + k$ translates $f(x)$ up or down

This translation is a "slide" straight up or down.
- if $k > 0$, the graph translates upward k units.
- if $k < 0$, the graph translates downward k units.

On a grid, you used the formula $(x,y) \rightarrow (x,y + k)$ to move a figure upward or downward. Keeping in mind that $y = f(x)$, we can write this formula as $(x, f(x)) \rightarrow (x, f(x) + k)$. Remember, you are adding the value of k to the y-values of the function.



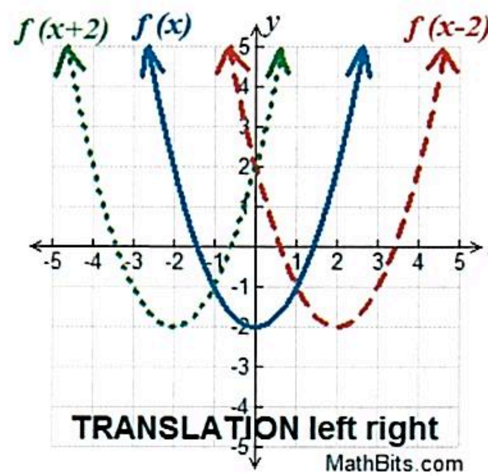
● Translation horizontally (left or right)

$f(x + k)$ translates $f(x)$ left or right

This translation is a "slide" left or right.
- if $k > 0$, the graph translates to the **left** k units.
- if $k < 0$, the graph translates to the **right** k units.

BEWARE

This one will not be obvious from the patterns you previously used when translating points.
A horizontal shift means that every point (x,y) on the graph of $f(x)$ is transformed to $(x - k, y)$ or $(x + k, y)$ on the graphs of $y = f(x + k)$ or $y = f(x - k)$ respectively.
Look carefully as this can be very confusing!



Hint: To remember which way to move the graph, set $(x + k) = 0$. The solution will tell you in which direction to move and by how much.

$f(x - 2)$: $x - 2 = 0$ gives $x = +2$, move right 2 units.
$f(x + 3)$: $x + 3 = 0$ gives $x = -3$, move left 3 units.

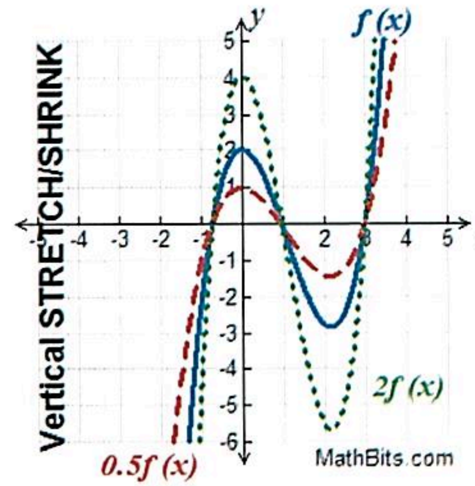
Dilations of Functions: $kf(x)$ and $f(kx)$

● **Vertical Stretch** or **Compression (Shrink)**

$k f(x)$ stretches/shrinks $f(x)$ vertically

A **vertical stretching** is the stretching of the graph away from the x-axis
A **vertical compression** (or **shrinking**) is the squeezing of the graph toward the x-axis.
- if $k > 1$, the graph of $y = k \cdot f(x)$ is the graph of $f(x)$ **vertically stretched** by multiplying each of its y-coordinates by k.
- if $0 < k < 1$ (*a fraction*), the graph is $f(x)$ **vertically shrunk (or compressed)** by multiplying each of its y-coordinates by k.

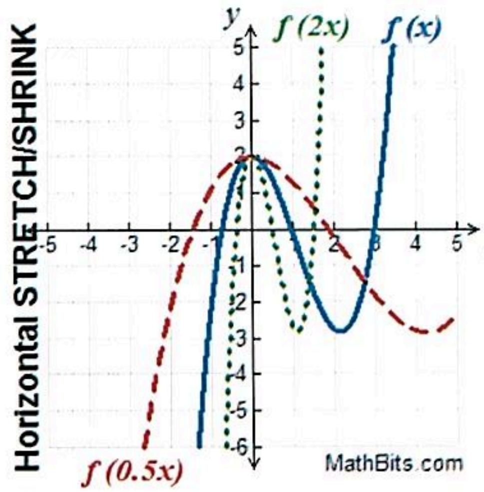
• if k should be negative, the vertical stretch or shrink is followed by a reflection across the x-axis.
Notice that the "roots" on the graph stay in their same positions on the x-axis. The graph gets "taffy pulled" up and down from the locking root positions. The y-values change.



"Multiply y-coordinates"
(x, y) becomes (x, ky)
"vertical dilation"

● **Horizontal Stretch** or **Compression (Shrink)**
$f(kx)$ stretches/shrinks $f(x)$ horizontally

A **horizontal stretching** is the stretching of the graph away from the y-axis
A **horizontal compression** (or **shrinking**) is the squeezing of the graph toward the y-axis.
• if $k > 1$, the graph of $y = k \cdot f(x)$ is the graph of $f(x)$ **horizontally shrunk (or compressed)** by dividing each of its x-coordinates by k.
• if $0 < k < 1$ (*a fraction*), the graph is $f(x)$ **horizontally stretched** by dividing each of its x-coordinates by k.
• if k should be negative, the horizontal stretch or shrink is followed by a reflection in the y-axis.
Notice that the "roots" on the graph have now moved, but the y-intercept stays in its same initial position for all graphs. The graph gets "taffy pulled" left and right from the locking y-intercept. The x-values change.



"Divide x-coordinates"
(x, y) becomes (x/k, y)
"horizontal dilation"

Transformations of Function Graphs	
$-f(x)$	reflect $f(x)$ over the x-axis
$f(-x)$	reflect $f(x)$ over the y-axis
$f(x) + k$	shift $f(x)$ up k units
$f(x) - k$	shift $f(x)$ down k units
$f(x + k)$	shift $f(x)$ left k units

$f(x - k)$	shift $f(x)$ right k units
$k \cdot f(x)$	multiply y-values by k
$f(kx)$	divide x-values by k