

Name \_\_\_\_\_

Mrs. Roubos

Date \_\_\_\_\_

8R Period \_\_\_\_\_

## Analyzing Solutions

### I. Possible Solutions of One-Variable Equations

Until now, when you have solved equations, there has been only one solution. It is also possible for a one-variable equation to have no solutions or infinitely many solutions.

#### EXAMPLE 1 One-Variable Equations with No Solutions

Solve the equation, and interpret the result.

$$2x - 4 = 2(x - 1) + 2$$

$$2x - 4 = 2(x - 1) + 2$$

$$2x - 4 = 2x$$

$2x - 2 + 2$  is  $2x$ .

Add 4 to both sides.

$$\begin{array}{r} +4 \\ 2x - 4 = 2x \\ \hline 2x = 2x + 4 \end{array}$$

$$\begin{array}{r} -2x \\ 2x = 2x + 4 \\ \hline 0 = 2x + 4 \end{array}$$

Subtract  $2x$  from both sides.

$$\begin{array}{r} -2x \\ 0 = 2x + 4 \\ \hline 0 = 4 \end{array}$$

This is a false statement.

Because  $0 = 4$  is never a true statement, the equation can never be true for *any* value of  $x$ . There is no solution.

#### EXAMPLE 2 One-Variable Equations with Infinitely Many Solutions

Solve the equation, and interpret the result.

$$-4x + 3(x - 1) = -(x + 3)$$

$$-4x + 3(x - 1) = -(x + 3)$$

$$-4x + 3x - 3 = -x - 3$$

Apply the Distributive Property.

$$-x - 3 = -x - 3$$

Simplify.

$$\begin{array}{r} +3 \\ -x - 3 = -x - 3 \\ \hline -x = -x \end{array}$$

Add 3 to both sides.

$$-x = -x$$

$$\begin{array}{r} +x \\ -x = -x \\ \hline 0 = 0 \end{array}$$

Add  $x$  to both sides.

$$0 = 0$$

This is a true statement.

Because  $0 = 0$  is always a true statement, the equation is always true for *any* value of  $x$ . There are *infinitely many* solutions.

Result	What does this mean	How many solutions
$x = a$	When the value of $x$ is $a$ , the equation is a true statement	1
$a = a$	Any value of $x$ makes the equation a true statement.	Infinitely many
$a = b$	There is no value of $x$ that makes the equation true.	0

Solve the equation, and interpret the result. Tell whether each equation has one, zero, or infinitely many solutions.

1)  $6 + 3x = x - 8$

2)  $8x + 4 = 4(2x + 1)$

3)  $4x - 3 = 2x + 13$

4)  $4x - 5 = 2(2x - 1) - 3$

5)  $4x + 2 = 4x - 5$

6)  $3(x-1) - 2x = -2x + 3(x - 1)$

7)  $-(2x + 2) - 1 = -x - (x + 3)$

8)  $7x + 2(x - 2) = 3(3x + 4)$

9)  $4(x - 2) - 2x = 3x - (x - 2)$

10)  $9w + 1 = 9(w + 1)$