Name Key
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Date_____
8R Period_____

Analyzing Solutions

I. Possible Solutions of One-Variable Equations

Until now, when you have solved equations, there has been only one solution. It is also possible for a one-variable equation to have no solutions or infinitely many solutions.

EXAMPLE 1 One-Variable Equations with No Solutions

Solve the equation, and interpret the result.

$2x - 4 = 2(x - 1) + 2$

$$\begin{aligned} 2x - 4 &= 2(x - 1) + 2 \\ 2x - 4 &= 2x \qquad \text{2x − 2 + 2 is 2x.} \\ +4 &\quad +4 \qquad \text{Add 4 to both sides.} \\ 2x &= 2x + 4 \\ -2x &= -2x \qquad \text{Subtract 2x from both sides.} \\ 0 &\neq 4 \qquad \text{This is a false statement.} \end{aligned}$$

Because $0 = 4$ is never a true statement, the equation can never be true for *any value* of x. There is no solution.

EXAMPLE 2 One-Variable Equations with Infinitely Many Solutions

Solve the equation, and interpret the result.

$-4x + 3(x - 1) = -(x + 3)$

$$\begin{aligned} -4x + 3(x - 1) &= -(x + 3) \\ -4x + 3x - 3 &= -x - 3 \qquad \text{Apply the Distributive Property.} \\ -x - 3 &= -x - 3 \qquad \text{Simplify.} \\ +3 &\quad +3 \qquad \text{Add 3 to both sides.} \\ -x &= -x \\ +x &\quad +x \qquad \text{Add x to both sides.} \\ 0 &= 0 \qquad \text{This is a true statement.} \end{aligned}$$

Because $0 = 0$ is always a true statement, the equation is always true for *any value* of x. There are *infinitely many* solutions.

Result	What does this mean	How many solutions
x = a ex x = 5	When the value of x is a, the equation is a true statement	1
a = a 6 = 6	Any value of x makes the equation a true statement.	Infinitely many
a = b 10 ≠ 6	There is no value of x that makes the equation true.	0, NO

☆ Make sure to Move the variable before the constant ☆

Solve the equation, and interpret the result. Tell whether each equation has one, zero, or infinitely many solutions.

1) $6 + 3x = x - 8$

$-x \quad -x$

$6 + 2x = -8$

$-6 \quad -6$

$\frac{2x}{2} = \frac{-14}{2}$

$x = -7$ one solution

2) $8x + 4 = 4(2x + 1)$

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$8x + 4 = 8x + 4$

$-8x \quad -8x$

$4 = 4$

Infinitely many

3) $4x - 3 = 2x + 13$

$-2x \quad -2x$

$2x - 3 = 13$

$+3 \quad +3$

$\frac{2x}{2} = \frac{16}{2}$

$x = 8$ one solution

4) $4x - 5 = 2(2x - 1) - 3$

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$4x - 5 = 4x - 2 - 3$

$4x - 5 = 4x - 5$

$-4x \quad -4x$

$-5 = -5$

Infinitely many

5) $4x + 2 = 4x - 5$

$-4x \quad -4x$

$2 \neq -5$

No solutions

6) $3(x-1) - 2x = -2x + 3(x - 1)$

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$3x - 3 - 2x = -2x + 3x - 3$

$3x - 2x - 3 = 1x - 3$

$1x - 3 = 1x - 3$

$-1x \quad -1x$

$-3 = -3$

Infinitely many

7) $-1(2x + 2) - 1 = -x - 1(x + 3)$

D C M S

$-2x - 2 - 1 = -1x - 1x - 3$

$-2x - 3 = -2x - 3$

$+2x \quad +2x$

$-3 = -3$

Infinitely many ∞

8) $7x + 2(x - 2) = 3(3x + 4)$

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$7x + 2x - 4 = 9x + 12$

$9x - 4 = 9x + 12$

$-9x \quad -9x$

$-4 \neq 12$

No solutions

9) $4(x - 2) - 2x = 3x - (x - 2)$

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$4x - 8 - 2x = 3x - x + 2$

$4x - 2x - 8 = 2x + 2$

$-2x \quad -2x$

$-8 \neq 2$

NONE

10) $9w + 1 = 9(w + 1)$

$9w + 1 = 9w + 9$

$-9w \quad -9w$

$1 \neq 9$

Zero solutions