Name Key Date

Lesson 15: Solving Systems of Equations Graphically

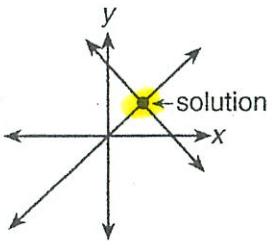
A **system of linear equations** consists of two or more linear equations. A **solution to a system of linear equations** is any ordered pair that is a solution to each equation in the system. In this lesson, you will solve systems of linear equations by graphing.

To solve a system of equations graphically, graph each of the equations in the system. The solution(s), if any, will be the ordered pair(s) of the point(s) of intersection of all the graphs.

There are three types of systems of linear equations. Each type has a different number of solutions. A **consistent system** has exactly one solution. An **inconsistent system** has no solution. A **dependent system** has an infinite number of solutions. You can determine the type of system by comparing the slopes and *y*-intercepts of the equations in the system.

Consistent System

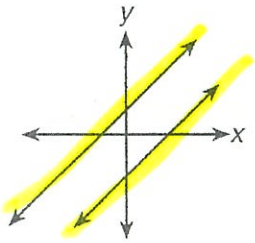
- exactly one solution
- different slopes, different y-intercepts



Ex
$x = 5$
1 solution

Inconsistent System

- no solution
- same slope, different *y*-intercepts

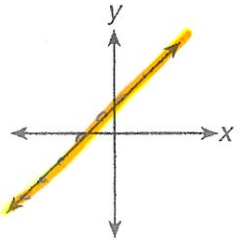


Ex
$5 \neq 6$
No solution

Dependent System

- infinite number of solutions
- same slope, same *y*-intercept

all ordered pairs



Ex
$5 = 5$
Infinitely many
Really 2 lines

CCSS: 8.EE.8a, 8.EE.8b

Consistent System

A consistent system will have exactly one solution. The graphs of the lines in the system will intersect at one point. The solution will be the ordered pair of this point of intersection. In this type of system, the slopes of the graphs are different.

Example

Solve the following system of equations graphically.

$2x - y = 4$
$x + 4y = 20$

Step 1: **Write each equation in the system in slope-intercept form.**

$2x - y = 4$
$-y = -2x + 4$
$y = 2x - 4$

slope: 2
y-intercept: -4

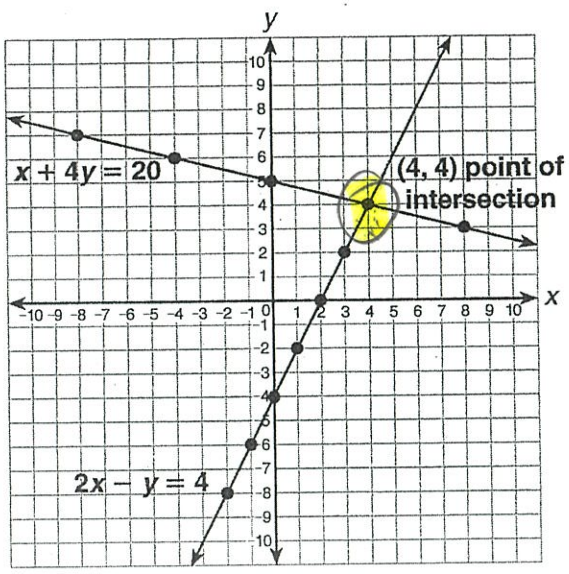
Different slope
Different y-int

$x + 4y = 20$
$4y = -x + 20$
$y = -\frac{1}{4}x + 5$

slope: $-\frac{1}{4}$
y-intercept: 5

By looking at the slopes, you should be able to determine the type of system of equation that is graphed. Since the slopes are different, this system is consistent and the graph will show a pair of intersecting lines.

Step 2: **Plot each *y*-intercept. Then find other points using the slope. Draw the graph of each equation on the same coordinate plane.**



The two lines appear to intersect at (4, 4). This is a possible solution, but you need to check to see that it is a solution to both equations in the system.

Step 3: **Check the point of intersection.**

Substitute the values for the variables into each of the original equations to see if the ordered pair is a solution to both.

Check (4, 4): $2x - y = 4$

$2(4) - (4) \stackrel{?}{=} 4$

$8 - 4 \stackrel{?}{=} 4$

$4 = 4$ ✓

$x + 4y = 20$

$(4) + 4(4) \stackrel{?}{=} 20$

$4 + 16 \stackrel{?}{=} 20$

$20 = 20$ ✓

Since the ordered pair is a solution to both equations in the system, the solution to the system is (4, 4). This is a consistent system of linear equations.

Inconsistent System

An inconsistent system will have no solution. The graphs of the lines in the system will be parallel. In this type of system, the slopes of the graphs in the system are the same and the *y*-intercepts of the graphs are different.

Example

Solve the following system of equations graphically.

$x + 4y = 12$
$3x + 12y = -24$

Write each equation in the system in slope-intercept form. Then graph each of them on the same coordinate plane.

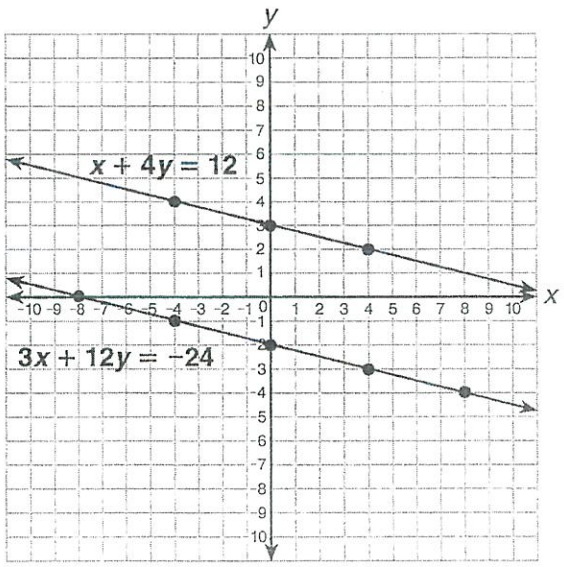
$x + 4y = 12$
$4y = -x + 12$
$y = -\frac{1}{4}x + 3$

$3x + 12y = -24$
$12y = -3x - 24$
$y = -\frac{1}{4}x - 2$

slope: $-\frac{1}{4}$ Same Slope slope: $-\frac{1}{4}$

y-intercept: 3 diff y-int. *y*-intercept: -2

The slopes are the same and the *y*-intercepts are different, so the graph will show a pair of parallel lines.



Since parallel lines do not intersect, the system has no solution. This is an inconsistent system of linear equations.

Dependent System

A dependent system will have an infinite number of solutions. The graphs of the lines in the system will be the same, so any point on the graph of one equation is also on the graph of the other equation. In this type of system, both the slopes and *y*-intercepts of the graphs are the same.

Example

Solve the following system of equations graphically.

$4x - y = -9$
$-12x + 3y = 27$

Write each equation in the system in slope-intercept form. Then, graph each of them on the same coordinate plane.

$4x - y = -9$
$-y = -4x - 9$
$y = 4x + 9$

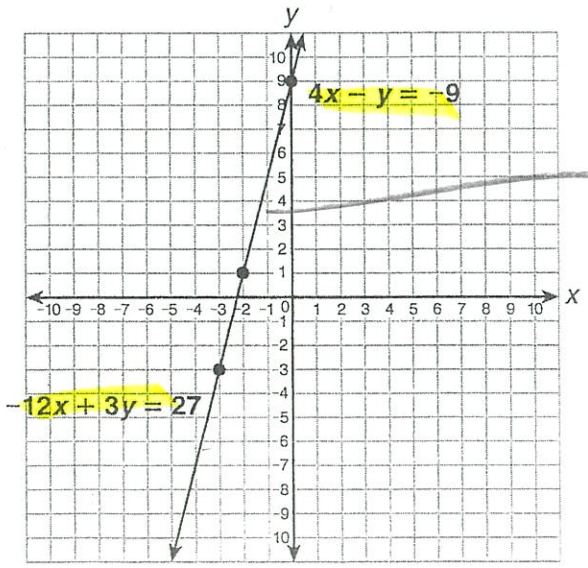
slope: 4
y-intercept: 9

Same slope
Same y-intercept

$-12x + 3y = 27$
$3y = 12x + 27$
$y = 4x + 9$

slope: 4
y-intercept: 9

The slopes and *y*-intercepts are the same, so the graph will show one line.



really
2 lines

Since the graphs of the equations are the same line, the system has an infinite number of solutions. The solutions are all the ordered pairs that are solutions of either $4x - y = -9$ or $-12x + 3y = 27$. This is a dependent system of linear equations.

Estimate Solutions by Graphing

Some solutions of a system of equations will not include integer values. In those cases, you can estimate the approximate solution by identifying a nearby point on the graph.

Example

Solve the following system of equations graphically.

$x + 2y = 1$
$-8x + 8y = -35$

Write each equation in the system in slope-intercept form. Then, graph each of them on the same coordinate plane.

$x + 2y = 1$
$2y = -x + 1$
$y = -\frac{1}{2}x + \frac{1}{2}$

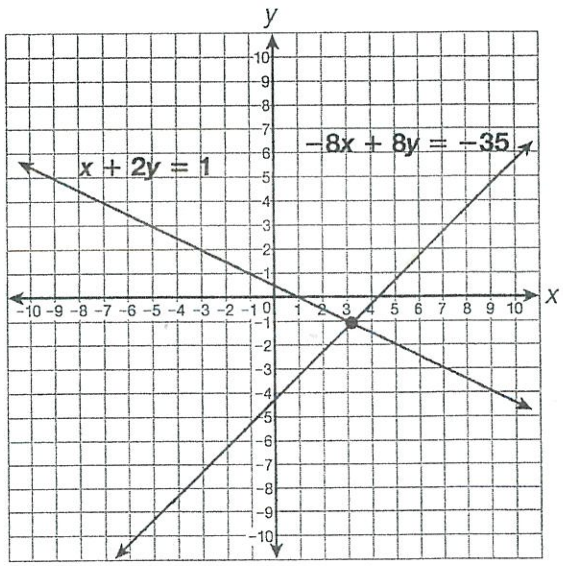
slope: $-\frac{1}{2}$

y-intercept: $\frac{1}{2}$

$-8x + 8y = -35$
$8y = 8x - 35$
$y = x - \frac{35}{8}$

slope: 1

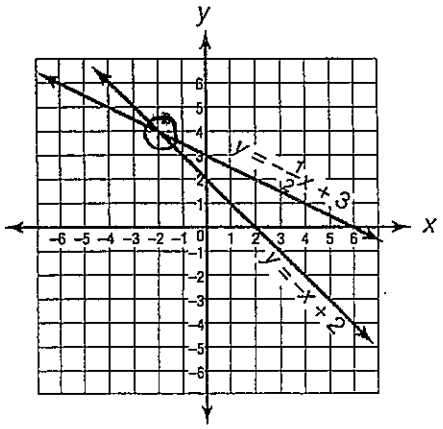
y-intercept: $-4\frac{3}{8}$



The solution to this system of equations does not include integer values. However, the solution is close to the point (3, −1) on the graph. Therefore, you can estimate that the solution is about (3, −1). This is a consistent system of linear equations.

Name _____ Systems Review

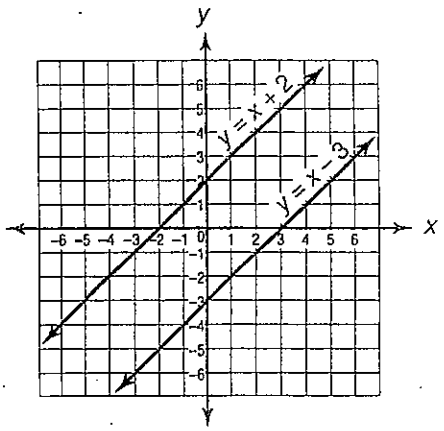
2 What is the solution of the system of linear equations graphed below?



A (4,−2)
B (0,2)
C (0,3)
D (−2,4)

✗ Consistent

3 What is the solution of the system of linear equations graphed below?



F all ordered pairs on both lines
G There is no solution.
H (0,2)
J (0,−3)

✗ Inconsistent

4 Roland has to find the solution of this system of linear equations.

$\frac{2y}{2} = \frac{4x}{2} - \frac{2}{2}$ $y = 2x - 1$ m: 2 B: −1

$\frac{3y}{3} = \frac{6x}{3} - \frac{3}{3}$ $y = 2x - 1$ m: 2 B: −1

Without graphing, what is the solution? [Hint: Divide both sides of the first equation by 2, and divide both sides of the second equation by 3. Then compare the equations.]

A all ordered pairs on both lines
B There is no solution.
C (0,−2)
D (0,−3)

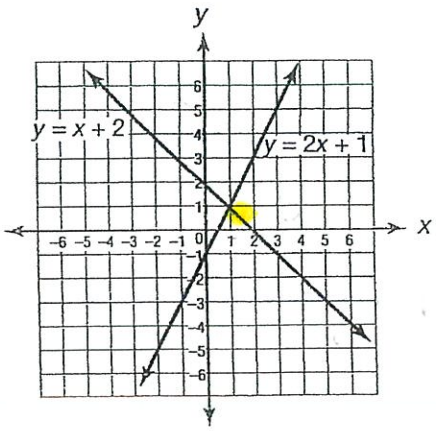
✗ Dependent

1 If two lines are parallel, what do you know about their ~~equations~~? solutions

F They have all their solutions in common.
G They have no solutions in common.
H They have exactly one solution in common.
J They have exactly two solutions in common.

✗ Inconsistent

5 What is the solution of the system of linear equations graphed below?



A (2,−1)

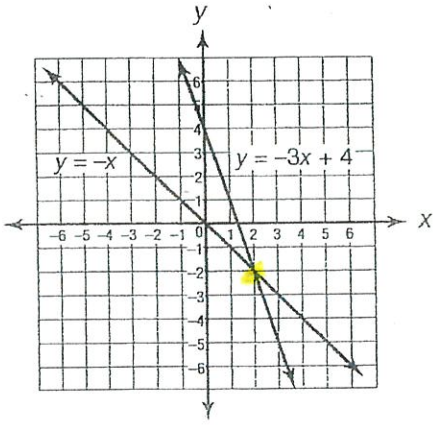
B (2,2)

C (1,1)

D (1,2)

Consistent

6 What is the solution of the system of linear equations graphed below?



F (2,−2)

G (−2,2)

H (2,2)

J (0,4)

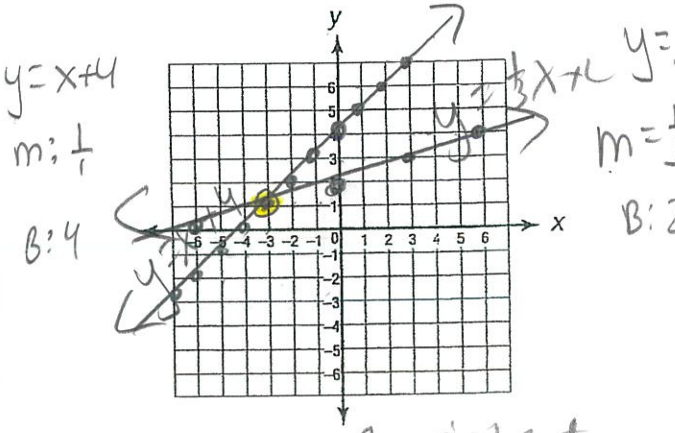
Consistent

7 What is the solution of this system of linear equations?

$y = x + 4$ m: $\frac{1}{1}$ → B: 4

$y = \frac{1}{3}x + 2$ m: $\frac{1}{3}$ → B: 2

Use the grid to sketch the graphs.



None: Consistent

A (−2,2)

B (−3,1)

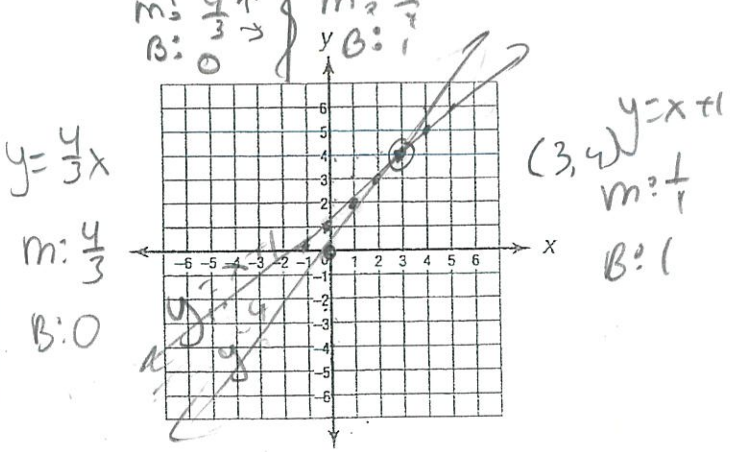
C (−1,3)

D (4,2)

8 What is the solution of this system of linear equations? Write your answer.

$y = \frac{4}{3}x$ $y = x + 1$

Use the grid to sketch the graphs.



Answer (3,4)

Consistent

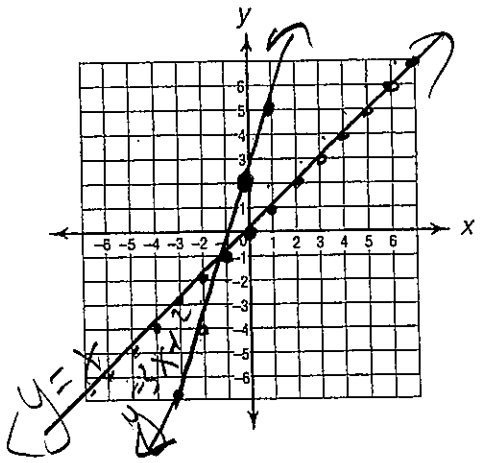
9

What is the solution of this system of linear equations? Write your answer.

$y = 3x + 2$ m: 3 $\frac{3}{1}\uparrow\rightarrow$ B: 2

$y = x$ m: $\frac{1}{1}$ B: 0

Use the grid to sketch the graphs.



$y = 3x + 2$

m: $\frac{3}{1}$

B: 2

$y = x$

m: 1

B: 0

Answer: (-1, -1)

Name: Consistent

Non-Constructed Response

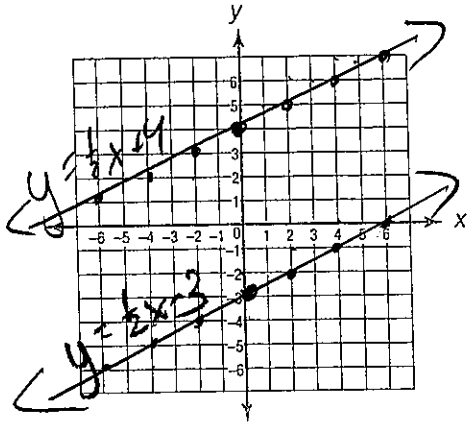
10

Part A

Graph this system of equations on the coordinate grid below.

$y = \frac{1}{2}x + 4$ m: $\frac{1}{2}\rightarrow$ B: 4

$y = \frac{1}{2}x - 3$ m: $\frac{1}{2}$ B: -3



$y = \frac{1}{2}x + 4$

m: $\frac{1}{2}$

B: 4

$y = \frac{1}{2}x - 3$

m: $\frac{1}{2}$

B: -3

Part B

What is the solution of the system?

NO Solution

Part C Name: Inconsistent

11)

Part A: Graph the system of equations on the coordinate grid below.

$y = x - 6$ m: $\frac{1}{1}$ B: -6

$\frac{2y}{2} = \frac{2x}{2} - \frac{12}{2}$

$y = x - 6$ m: $\frac{1}{1}$ B: -6

$y = x - 6$

m: $\frac{1}{1}$

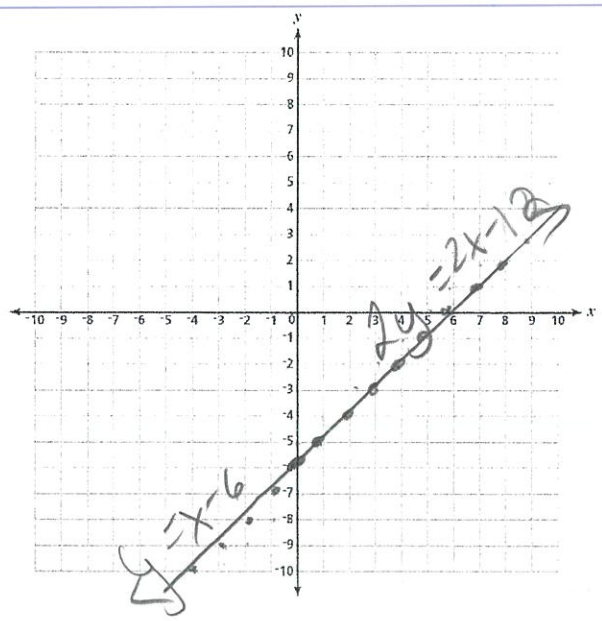
B: -6

$\frac{2y}{2} = \frac{2x}{2} - \frac{12}{2}$

$y = x - 6$

m: $\frac{1}{1}$

B: -6



Part B: What is the solution of the system? Infinite #

Part C: What is the name of the type of system? Dependent

12)

Part A: Graph the system of equations on the coordinate grid below.

$y = \frac{1}{2}x - 4$ m: $\frac{1}{2}$ B: -4

$\frac{2y}{2} = \frac{x}{2} + \frac{6}{2}$

$y = \frac{1}{2}x + 3$ m: $\frac{1}{2}$ B: 3

$y = \frac{1}{2}x - 4$

m: $\frac{1}{2}$

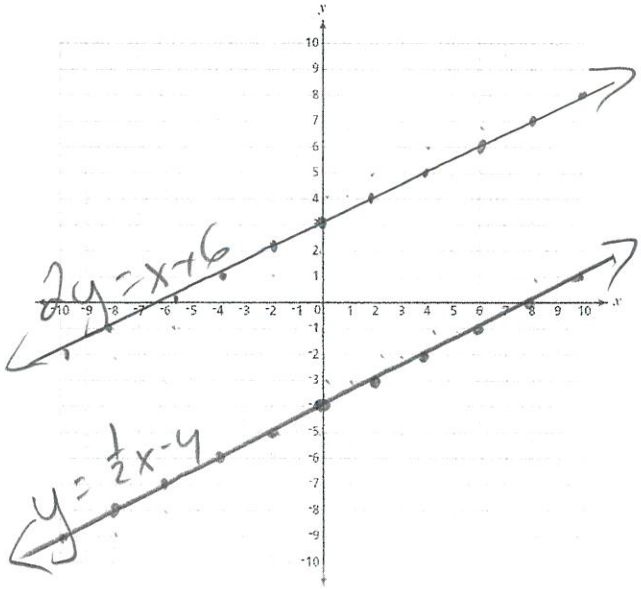
B: -4

$\frac{2y}{2} = \frac{x}{2} + \frac{6}{2}$

$y = \frac{1}{2}x + 3$

m: $\frac{1}{2}$

B: 3



Part B: What is the solution of the system? No Solutions

Part C: What is the name of the type of system? Inconsistent

13)

Part A: Graph the system of equations on the coordinate grid below.

$y = 2x - 7$ $\quad m: 2 \tfrac{2}{1}$ $\quad B: -7$

$y = -1$ $\quad m: 0$ $\quad B: -1$

$y = 2x - 7$

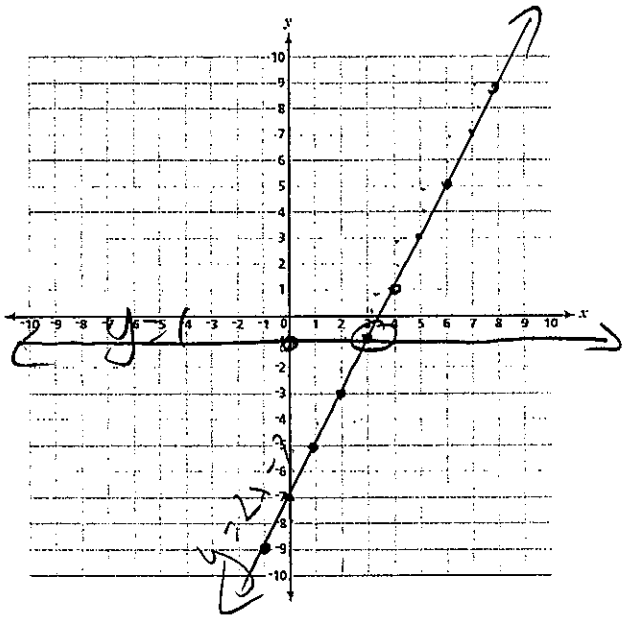
$m: \frac{2}{1}$

$B: -7$

$y = -1$

$m: 0$

$B: -1$



Part B: What is the solution of the system? $(3, -1)$

Part C: What is the name of the type of system? Consistent