

## Relations and Functions

8.A.17 Define and use correct terminology when referring to function (domain and range)

8.A.18 Determine if a relation is a function

An **ordered pair** names the location of a point on a coordinate grid. The  $x$ -coordinate (the first coordinate) tells you how far to the right or left of 0 the point is, and the  $y$ -coordinate tells you how far up or down from 0 the point is.

A **relation** is a set of ordered pairs, for example,  $\{(-4,3), (4,9), (5,12)\}$ .

The **domain** of a relation is the set of  $x$ -values of the relation. The **range** of a relation is the set of  $y$ -values of the relation. For the relation shown above, the domain is  $\{-4, 4, 5\}$  and the range is  $\{3, 9, 12\}$ .

A **function** is a relation in which each element of the domain corresponds to one and only one element of the range. In terms of ordered pairs, a function is a relation in which no two ordered pairs have the same  $x$ -coordinate. The relation above has no two  $x$ -coordinates that are the same, so that relation is a function.

## EXAMPLE 1

Is the following relation a function? Why or why not?

$$\{(1,5), (2,10), (3,15), (4,20)\}$$

## STRATEGY

Use the definition of function.

STEP 1 List the  $x$ -coordinates.

1, 2, 3, 4

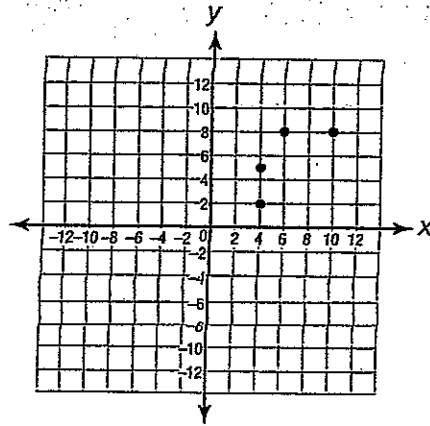
STEP 2 Do any of the  $x$ -coordinates repeat?

Each  $x$ -coordinate appears only once. Another way of stating this is that each  $x$ -coordinate (domain value) is paired with exactly one  $y$ -coordinate (range value).

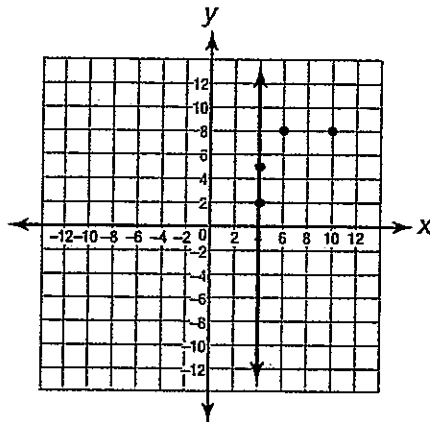
## SOLUTION

The relation is a function because each member of the domain is paired with exactly one member of the range.

Relations and functions are sets of ordered pairs, so they can be graphed on a coordinate plane. Here is a graph of the relation  $\{(4,2), (4,5), (6,8), (10,8)\}$ .



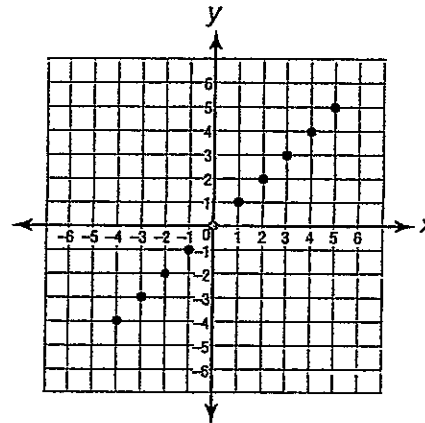
If the graph of a relation has more than one  $y$ -coordinate for an  $x$ -coordinate, then the relation is not a function. There is an easy way to check if there is more than one  $y$ -coordinate for each  $x$ -coordinate. If you can draw a vertical line through more than one point on the graph, then the relation is not a function.



A vertical line can be drawn through two points, so the relation  $\{(4,2), (4,5), (6,8), (10,8)\}$  is not a function.

**EXAMPLE 2**

Does the graph show a function? If so, why?

**STRATEGY**

Use the vertical line test.

It is not possible to draw a vertical line through more than one point.

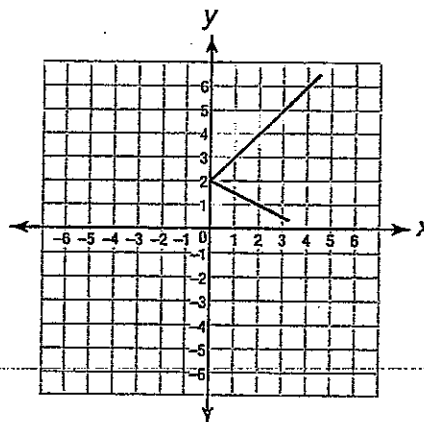
**SOLUTION**

The graph shows a function because it is impossible to draw a vertical line through more than one point.

The relation in Example 2 has a finite (countable) number of points. The vertical line test can also be used for relations that consist of an infinite number of points.

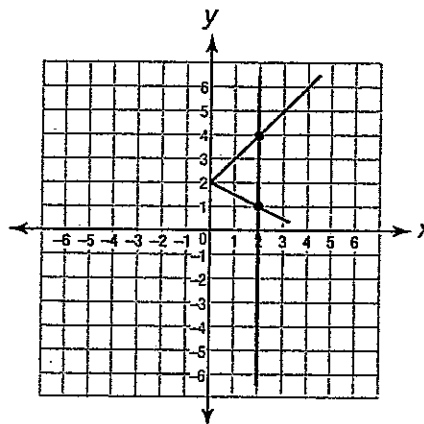
**EXAMPLE 3**

The following graph shows a relation with an infinite number of points. Does the graph represent a function? Why or why not?

**STRATEGY**

Use the vertical line test.

You can draw a vertical line through two points at many locations. One example would be the points at (2,1) and (2,4).



**SOLUTION**

The graph is not a function because it is possible to draw a vertical line that intersects the graph in more than one point.



Is this relation a function?

$\{(-2,4), (-1,1), (0,0), (1,1), (2,4)\}$

Let's check it out.

The domain of the relation is \_\_\_\_\_.

The range of the relation is \_\_\_\_\_.

Do any of the elements of the domain repeat? \_\_\_\_\_

Explain why the relation is or is not a function. \_\_\_\_\_

\_\_\_\_\_

# Sample Test Questions

1 What is the range of this relation?

$$\{(1,2), (3,4), (5,6), (7,8), (7,10)\}$$

- A {1, 3, 5, 7, 8}
- B {5, 6, 7, 8, 10}
- C {2, 4, 6, 7, 8}
- D {2, 4, 6, 8, 10}

2 Which relation is a function?

- F  $\{(1,3), (2,2), (3,3), (3,4)\}$
- G  $\{(10,15), (20,40), (20,45), (30,50)\}$
- H  $\{(12,28), (14,28), (16,30), (18,32)\}$
- J  $\{(22,11), (24,12), (26,12), (26,13)\}$

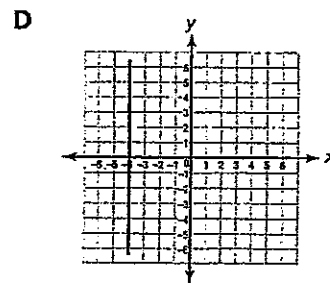
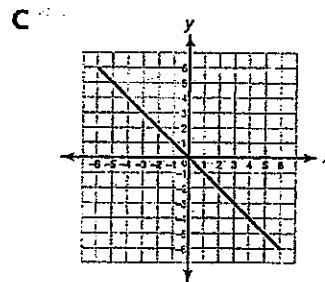
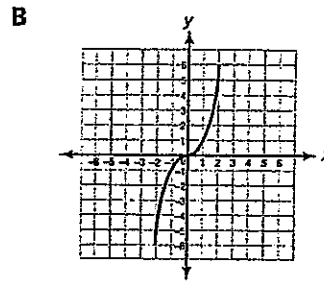
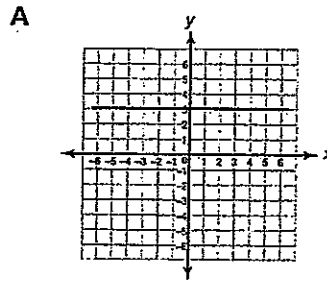
3 Which relation is a function?

- A  $\{(-1,1), (2,8), (-1,-1), (-2,-8)\}$
- B  $\{(1,2), (2,8), (3,18), (2,32)\}$
- C  $\{(-3,9), (-2,4), (2,4), (3,9)\}$
- D  $\{(1,5), (2,10), (-2,10), (1,15)\}$

4 Which relation is not a function?

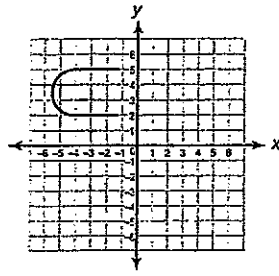
- F  $\{(\frac{1}{2}, \frac{1}{3}), (\frac{1}{4}, \frac{1}{5}), (\frac{1}{6}, \frac{1}{7}), (\frac{1}{8}, \frac{1}{9})\}$
- G  $\{(\frac{1}{2}, \frac{2}{2}), (\frac{1}{3}, \frac{3}{2}), (\frac{2}{3}, \frac{2}{3}), (\frac{3}{3}, \frac{4}{3})\}$
- H  $\{(\frac{1}{2}, \frac{1}{5}), (\frac{3}{2}, \frac{1}{6}), (\frac{4}{2}, \frac{1}{7}), (\frac{5}{2}, \frac{1}{8})\}$
- J  $\{(\frac{1}{2}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{8}), (\frac{4}{8}, \frac{1}{16}), (\frac{4}{16}, \frac{1}{32})\}$

5 Which graph below does not show a function?

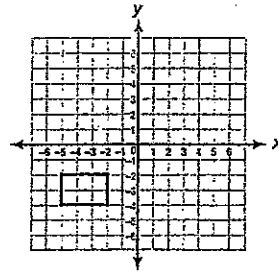


6 Which graph shows a function?

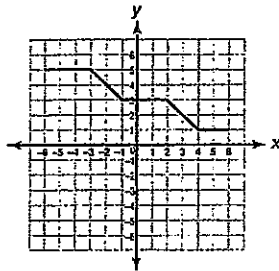
F



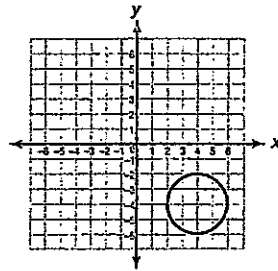
H



G



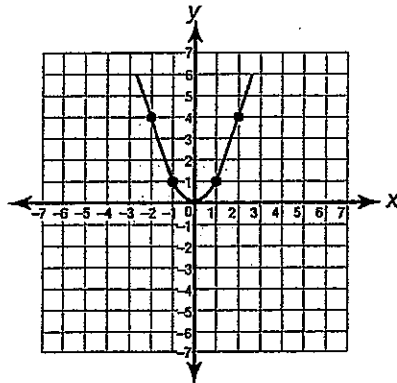
J



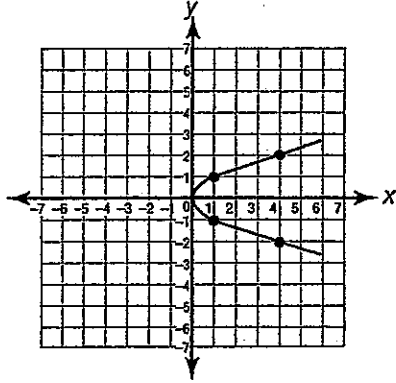
7

Which graph shows a function?

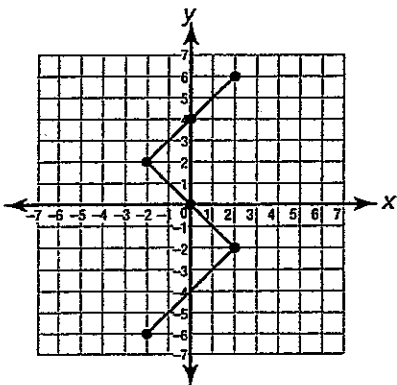
A



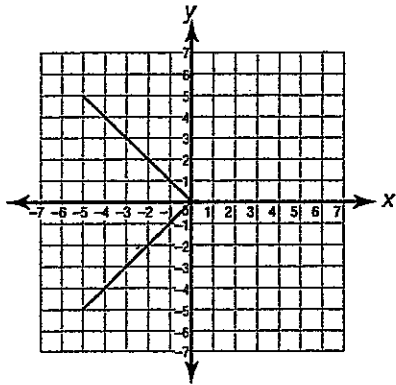
C



B



D



8 Write the domain of this relation.

$\{(5,25), (6,36), (-5,25), (-6,36)\}$

Answer: \_\_\_\_\_

9 Write the range of this relation.

$\{(2,20), (2,-40), (-2,10), (-2,-30)\}$

Answer: \_\_\_\_\_

## Extended-Response Question

10

Part A

Determine if the relation below is a function. Explain your answer.

$\{(-3,8), (-2,3), (-1,0), (0,-1), (1,0), (2,3), (3,8)\}$

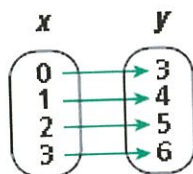
Answer: \_\_\_\_\_

Part B

How could you explain your answer to Part A using a graph?

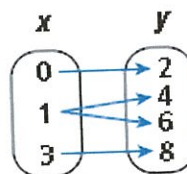
### Function

Each input gives only one output.



### Not a Function

One input gives more than one output.



## More Examples

Are the following examples of functions?

1)

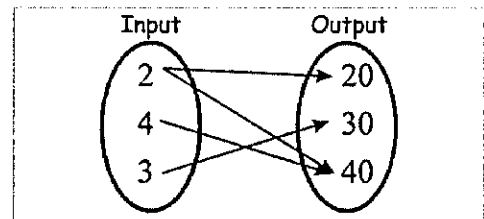
x	y
-1	-7
9	1
12	8
15	-7

2)

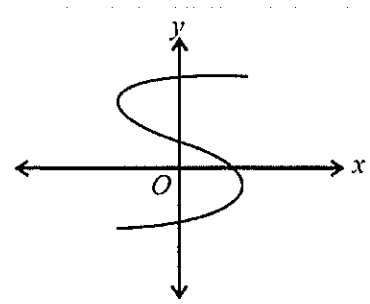
x	y
2	4
5	5
8	6
2	7

3)  $\{(1,3),(2,6),(1,3)\}$

4)



5)



6)

