Name Key
Mrs. Roumbos

Date_____
8R Period_____

Rate of Change
(slope)

A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

Delta change

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}} \qquad \frac{\Delta y}{\Delta x} \qquad \frac{y_2 - y_1}{x_2 - x_1}$$

The rates of change for a set of data may vary, or they may be constant.

**the dependent variable is y and the independent variable is x

same → different

Identifying Constant and Variable Rates of Change in Data

Determine whether the rates of change are constant or variable.

A

		+1	+2	+3	+2
x	0	1	3	6	8
y	0	4	8	8	6
		+4	+4	+0	−2

Find the differences between consecutive data points.

$\frac{4}{1} = 4$ $\frac{4}{2} = 2$ $\frac{0}{3} = 0$ $\frac{-2}{2} = -1$

Find each ratio of change in y to change in x.

The table shows nonlinear data. The rates of change are variable.

B

		+1	+3	+2	+1
x	0	1	4	6	7
y	1	2	5	7	8
		+1	+3	+2	+1

Find the differences between consecutive data points.

$\frac{1}{1} = 1$ $\frac{3}{3} = 1$ $\frac{2}{2} = 1$ $\frac{1}{1} = 1$

Find each ratio of change in y to change in x.

The table shows linear data. The rates of change are constant.

Examples

Determine whether the rates of change are constant or variable.

① $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

x	0	1	3	7	8
y	1	3	7	15	17

+2 +4 +8 +2

$\frac{2}{1} = 2$

$\frac{4}{2} = 2$

$\frac{8}{4} = 2$

$\frac{2}{1} = 2$

The rates of change are constant B/c all the slopes are the same

② $\frac{\Delta y}{\Delta x}$

x	2	4	5	6	7
y	2	6	7	13	14

+4 +1 +6 +1

$\frac{4}{2} = 2$

$\frac{1}{1} = 1$

$\frac{6}{1} = 6$

$\frac{1}{1} = 1$

The rates of change are variable B/c the slopes are different

Determine whether the rates of change are constant or variable.

③ $\frac{\Delta y}{\Delta x} =$

x	−1	0	3	5	9
y	1	3	6	10	4

+2 +3 +4 −6

$\frac{2}{1} = 2$

$\frac{3}{3} = 1$

$\frac{4}{2} = 2$

$\frac{-6}{4} = -\frac{3}{2}$

The rate of change are variable B/c the slopes are different

④ $\frac{\Delta y}{\Delta x}$

x	2	4	6	7	8
y	8	4	0	−2	−4

−4 −4 −2 −2

$\frac{-4}{2} = -2$

$\frac{-4}{2} = -2$

$\frac{-2}{1} = -2$

$\frac{-2}{1} = -2$

The rates of change are Constant B/c all the slopes are the same

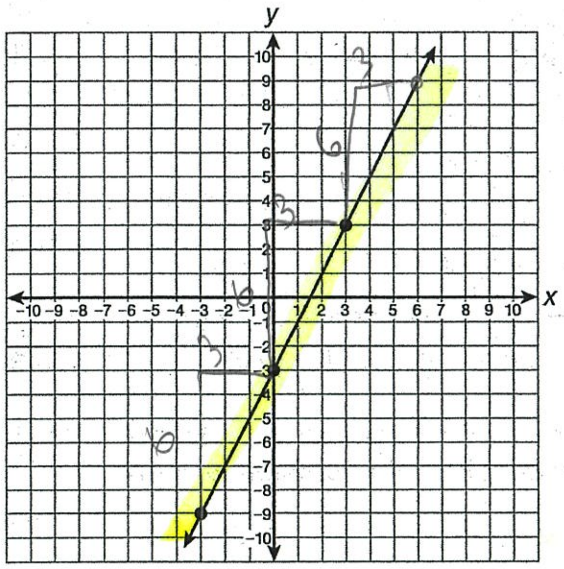
CCSS: 8.F.3

Lesson 20: Linear and Nonlinear Functions

When a function is represented by a graph, a line represents a linear function and a curve represents a nonlinear function.

Example

The following graph represents a linear function.



Constant slope / Rate of change

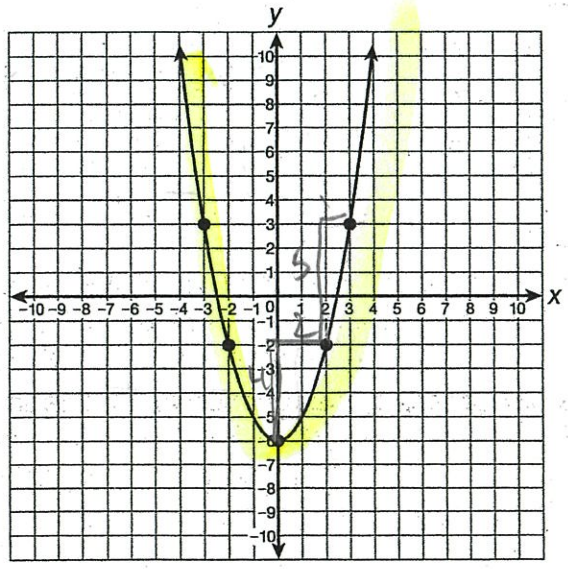
$\frac{6}{3} = 2$

$\frac{6}{3} = 2$

$\frac{6}{3} = 2$

Example

The following graph represents a nonlinear function.



Variable slope / rate of change

$\frac{4}{2} = 2$

$\frac{5}{1} = 5$

CCSS: 8.F.3

A linear function has a constant rate of change. A nonlinear function does not have a constant rate of change. A table can help show whether a function is linear or nonlinear. Examine the rate of change using the ordered pairs to see if there is a constant rate of change.

Example

The following table represents a function with the rate of change between rows shown on each side of the table. Is the relationship linear or nonlinear?

	x	y	
3	−6	−1	1
6	−3	0	2
9	3	2	3
12	12	5	4
	24	9	

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$$

Since the rate of change is constant, the function is linear.

Example

The following table represents a function with the rate of change between rows shown on each side of the table. Is the relationship linear or nonlinear?

	x	y	
2	−1	−13	10
4	1	−3	8
3	5	5	9
1	8	14	4
	9	18	

$$\frac{10}{2} \neq \frac{8}{4} \neq \frac{9}{3} \neq \frac{4}{1}$$

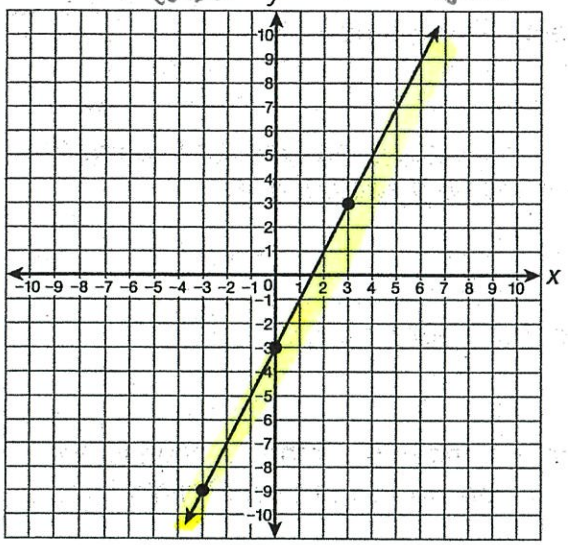
Since the rate of change is not constant, the function is nonlinear.

CCSS: 8.F.3

Practice

Directions: For questions 1 and 2, determine whether the given graph or table represents a linear or nonlinear function.

1. Constant ↓ y ↘ Variable



$\frac{6}{3} = 2$

$\frac{6}{3} = 2$

linear or nonlinear function? linear B/C it is a straight line and it has a constant rate of change

2. $\frac{\Delta y}{\Delta x}$

x	y	
−4	8	−2
−2	6	−4
0	2	−4
2	−2	−4
4	−4	

+2, +2, +2, +2

$\frac{-2}{2} = -1$

$\frac{-4}{2} = -2$ } variable

$\frac{-4}{2} = -2$

$\frac{-2}{2} = 1$

linear or nonlinear function? NON-linear B/C the rates of change are variable (NOT constant)

Directions: For questions 3 and 4, determine whether the given table represents a linear function. If it does, write the equation of the function in slope-intercept form.

3.

x	y
−1	1
1	1
3	3
5	5
7	7

$\frac{\Delta y}{\Delta x}$ 12 +2 +2 +2 | +0 +2 +2 +2 | $\frac{0}{2}=0$ $\frac{2}{2}=1$ $\frac{2}{2}=1$ $\frac{2}{2}=1$

NON-linear B/C the rates of change are variable

4.

x	y
8	−3
16	−4
24	−5
32	−6
48	−8

$\frac{\Delta y}{\Delta x}$ +8 +8 +8 +16 | −1 −1 −1 −2 | $\frac{-1}{8}=\frac{-1}{8}$ $\frac{-1}{8}=\frac{-1}{8}$ $\frac{-1}{8}=\frac{-1}{8}$ $\frac{-2}{16}=\frac{-1}{8}$

linear B/C the rates of change are constant

5. If the following table represents a linear function, write an equation in slope-intercept form to represent the function.

x	y
3	10
−1	−2
5	16
−2	−5
0	1

$\frac{\Delta y}{\Delta x}$ +4 +6 −7 +2 | −12 +18 −21 +6 | $\frac{-12}{-4}=3$ $\frac{18}{6}=3$ $\frac{-21}{-7}=3$ $\frac{6}{2}=3$

y-intercept

$y = 3x + 1$

Explain how you were able to identify whether the table represents a linear or nonlinear function.

linear B/C the rate of change are constant

$y = x^3 + 4 \Rightarrow$ NL

$y = x^2 + 4 \rightarrow$ NL

$y = x + 4 \rightarrow$ Linear

$y = \frac{1}{x} + 4 \rightarrow$ NON-linear